4 GRADE

Mathematics Curriculum

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Decimal Fractions

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OVERVIEW

This 20-day module gives students their first opportunity to explore decimal numbers via their relationship to decimal fractions, expressing a given quantity in both fraction and decimal forms. Utilizing the understanding of fractions developed throughout Module 5, students apply the same reasoning to decimal numbers, building a solid foundation for Grade 5 work with decimal operations. Previously referred to as whole numbers, all numbers written in the base-ten number system with place value units that are powers of 10 are henceforth referred to as decimal numbers, a set which now includes tenths and hundredths (e.g., 1, 15, 248, 0.3, 3.02, and 24.35).

In Topic A, students use their understanding of fractions to explore tenths. At the opening of the topic, they use metric measurement to see tenths in relation to different whole units: centimeters, meters, kilograms, and liters. Students explore, creating and identifying tenths of various wholes, as they draw lines of specified length, identify the weight of objects, and read the level of liquid measurements. Students connect these concrete experiences pictorially as tenths are represented on the number line and with tape diagrams as pictured to the right. Students express tenths as decimal fractions and are introduced to decimal notation. They write statements of equivalence in unit, fraction, and decimal forms (e.g., 3 tenths = $\frac{3}{10}$ = 0.3) (**4.NF.6**). Next, students return to the use of metric measurement to investigate decimal fractions greater than 1. Using a centimeter ruler, they draw lines that measure, for example, $2\frac{4}{10}$ or $6\frac{8}{10}$ centimeters. Using the area model, students see that numbers containing a whole number and fractional part (i.e., mixed numbers) can also be expressed using decimal notation, provided that the fractional part can be converted to a decimal number (**4.NF.6**). Students use place value disks to represent the value of each digit in a decimal number. Just as they wrote whole numbers in expanded form using multiplication, students write the value of a decimal number in expanded form using fractions and decimals; for example, 2 ones 4 tenths = $2\frac{4}{10}$ = (2 × 1) + (4 × $\frac{1}{10}$) and 2.4 = (2 × 1) + (4 × 0.1). Additionally, students plot decimal numbers on the number line.
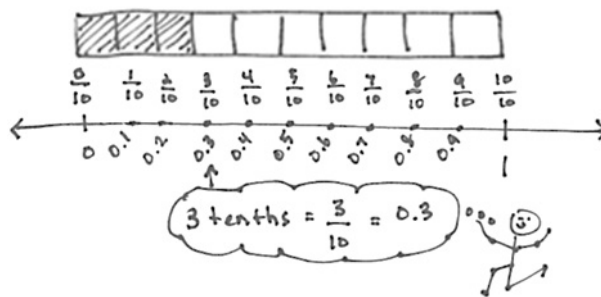
Students decompose tenths into 10 equal parts to create hundredths in Topic B. Through the decomposition of a meter, students identify 1 centimeter as 1 hundredth of a meter. As students count up by hundredths, they realize the equivalence of 10 hundredths and 1 tenth and go on to represent them as both decimal fractions and as decimal numbers (**4.NF.5**). Students use area models, tape diagrams, and place value disks on a place value chart to see and model the equivalence of numbers involving units of tenths and hundredths. They express the value of the number in both decimal and fraction expanded forms.

$$31\tfrac{46}{100} = (3 \times 10) + (1 \times 1) + \left(4 \times \tfrac{1}{10}\right) + \left(6 \times \tfrac{1}{100}\right)$$

$$31.46 = (3 \times 10) + (1 \times 1) + (4 \times 0.1) + (6 \times 0.01)$$

Close work with the place value chart helps students see that place value units are not symmetric about the decimal point—a common misconception that often leads students to mistakenly believe there is a *oneths* place. They explore the placement of decimal numbers to hundredths and recognize that the place value chart is symmetric about the ones column. This understanding helps students recognize

Symmetry with respect to the ones place

| hundred | ten | one | • | ten*th* | hundred*th* |

that, even as we move to the units on the right side of the decimal on the place value chart, a column continues to represent a unit 10 times as large as that of the column to its right. This understanding builds on the place value work done in Module 1 and enables students to understand that 3.2, for example, might be modeled as 3 ones 2 tenths, 32 tenths, or 320 hundredths. Topic B concludes with students using their knowledge of fraction equivalence to work with decimal numbers expressed in unit form, fraction form, and decimal form (**4.NF.6**).
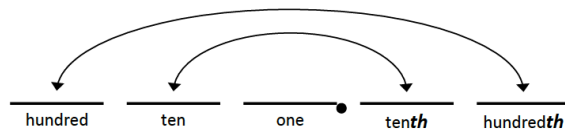
The focus of Topic C is comparison of decimal numbers (**4.NF.7**). To begin, students work with concrete representations of measurements. They see measurement of length on meter sticks, of mass using a scale, and of volume using graduated cylinders. In each case, students record the measurements on a place value chart and then compare them. They use their understanding of metric measurement and decimals to answer questions, such as, "Which is greater? Less? Which is longer?

Mass of Rice Bags (kilograms)

Rice Bag	ones	.	tenths	hundredths
A	0	.	1	0
B	0	.	6	5
C	0	.	7	
D	0	.	4	6

0.7 kg, 0.65 kg, 0.46 kg, 0.1 kg

Shorter? Which is heavier? Lighter?" Comparing the decimals in the context of measurement supports students' justification of their comparisons and grounds their reasoning, while at the same time setting them up for work with decimal comparison at a more concrete level. Next, students use area models and number lines to compare decimal numbers and use the <, >, and = symbols to record their comparisons. All of their work with comparisons at the pictorial level helps to eradicate the common misconception that is often made when students assume a greater number of hundredths must be greater than a lesser number of tenths. For example, when comparing 7 tenths and 27 hundredths, students recognize that 7 tenths is greater than 27 hundredths because, as in any comparison, one must consider the *size of the units*. Students go on to arrange mixed groups of decimal fractions in unit, fraction, and decimal forms in order from greatest to least or least to greatest. They use their understanding of different ways of expressing equivalent values to arrange a set of decimal fractions as pictured below.
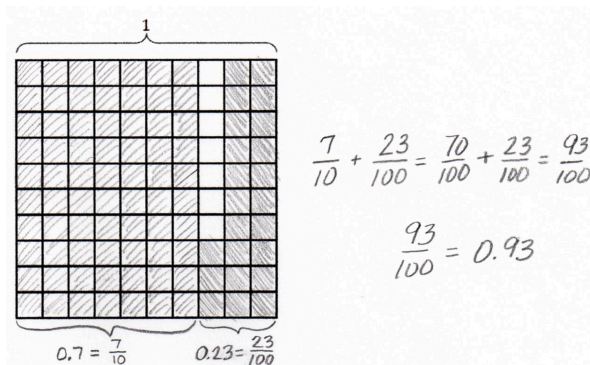
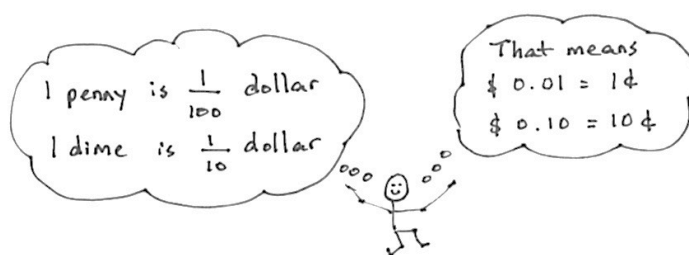

$$\tfrac{37}{100} < 0.5 < 1 \text{ and } 22 \text{ hundredths} < 1\tfrac{4}{10} < 1.54$$

Topic D introduces the addition of decimals by way of finding equivalent decimal fractions and adding fractions. Students add tenths and hundredths, recognizing that they must convert the addends to the same units (**4.NF.5**). The sum is then converted back into a decimal (**4.NF.6**). They use their knowledge of like denominators and understanding of fraction equivalence to do so. Students use the same process to add and subtract mixed numbers involving decimal units. They then apply their new knowledge to solve word problems involving metric measurements.

$$\frac{7}{10} + \frac{23}{100} = \frac{70}{100} + \frac{23}{100} = \frac{93}{100}$$

$$\frac{93}{100} = 0.93$$

$$0.7 = \frac{7}{10} \qquad 0.23 = \frac{23}{100}$$

Students conclude their work with decimal fractions in Topic E by applying their knowledge to the real-world context of money. They recognize 1 penny as $\frac{1}{100}$ dollar, 1 dime as $\frac{1}{10}$ dollar, and 1 quarter as $\frac{25}{100}$ dollar. They apply their understanding of tenths and hundredths to write given amounts of money

1 penny is $\frac{1}{100}$ dollar

1 dime is $\frac{1}{10}$ dollar

That means
$ 0.01 = 1¢
$ 0.10 = 10¢

in both fraction and decimal forms. To do this, students decompose a given amount of money into dollars, quarters, dimes, and pennies and express the amount as a decimal fraction and decimal number. Students then add various numbers of coins and dollars using Grade 2 knowledge of the equivalence of 100 cents to 1 dollar. Addition and subtraction word problems are solved using unit form, adding dollars and cents. Multiplication and division word problems are solved using cents as the unit (**4.MD.2**). The final answer in each word problem is converted from cents into a decimal using a dollar symbol for the unit. For example, *Jack has 2 quarters and 7 dimes. Jim has 1 dollar, 3 quarters, and 6 pennies. How much money do they have together? Write your answer as a decimal.*

Jack | 50¢ | 70¢

Jim | $1 | 75¢ | 6¢

They have $3.01 together.

1 dollar 20 cents + 1 dollar 81 cents

= 2 dollars 101 cents
       ^
      100 1

= 3 dollars 1 cent

= $3.01

Notes on Pacing for Differentiation

In Module 6, students explore decimal numbers for the first time by means of the decimal numbers' relationship to decimal fractions. Module 6 builds directly from Module 5 and is foundational to students' Grade 5 work with decimal operations. Therefore, it is not recommended to omit any lessons from Module 6.
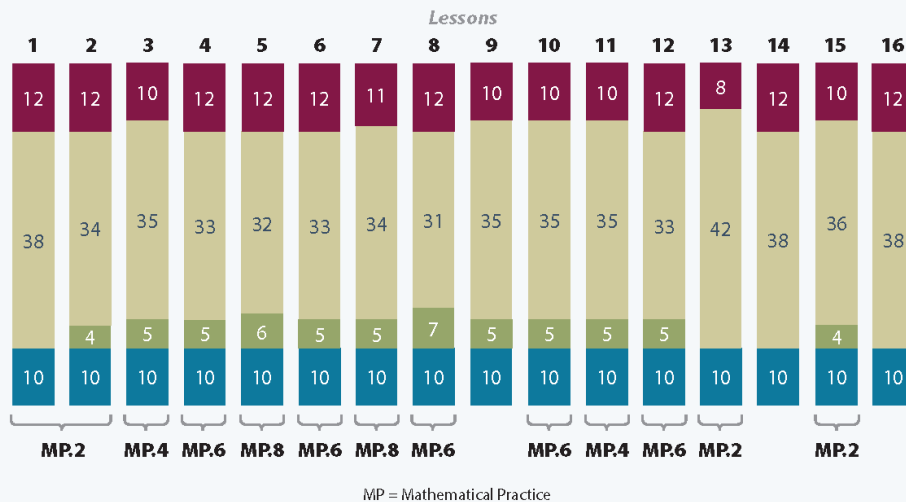
Distribution of Instructional Minutes

This diagram represents a suggested distribution of instructional minutes based on the emphasis of particular lesson components in different lessons throughout the module.

- ■ **Fluency Practice**
- ■ **Concept Development**
- ■ **Application Problems**
- ■ **Student Debrief**

Lessons

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Fluency Practice	12	12	10	12	12	12	11	12	10	10	10	12	8	12	10	12
Concept Development	38	34	35	33	32	33	34	31	35	35	35	33	42	38	36	38
Application Problems		4	5	5	6	5	5	7	5	5	5	5			4	
Student Debrief	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10

MP.2   MP.4 MP.6 MP.8 MP.6 MP.8 MP.6   MP.6 MP.4 MP.6 MP.2   MP.2

MP = Mathematical Practice

Focus Grade Level Standards

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5   Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)

4.NF.6   Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.7   Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. [1]

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Foundational Standards

2. MD.8 Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

3. NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80, 5×60) using strategies based on place value and properties of operations.

3. NF.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3. NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram.

   a. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3. NF.3 Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

   a. Recognize and generate simple equivalent fractions, (e.g., $1/2 = 2/4$, $4/6 = 2/3$). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

   b. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

3. MD.2 Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems [problems involving notions of "times as much"; see CCSS-M Glossary, Table 2]).

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[1]4.MD.1 is addressed in Modules 2 and 7; 4.MD.3 is addressed in Module 3.

Focus Standards for Mathematical Practice

MP.2   **Reason abstractly and quantitatively.** Throughout this module, students use area models, tape diagrams, place value disks, and number lines to represent decimal quantities. When determining the equivalence of a decimal fraction and a decimal number, students consider the units that are involved and attend to the meaning of the quantities of each. Further, students use metric measurement and money amounts to build an understanding of the decomposition of a whole into tenths and hundredths.

MP.4   **Model with mathematics.** Students represent decimals with various models throughout this module, including expanded form. Each of the models helps students to build understanding and to analyze the relationship and role of decimals within the number system. Students use a tape diagram to represent tenths and then to decompose one-tenth into hundredths. They use place value disks and a place value chart to extend their understanding of place value to include decimal fractions. Further, students use a place value chart along with the area model to compare decimals. A number line models decimal numbers to the hundredths.

MP.6   **Attend to precision.** Students attend to precision as they decompose a whole into tenths and tenths into hundredths. They also make statements such as 5 ones and 3 tenths equals 53 tenths. Focusing on the units of decimals, students examine equivalence, recognize that the place value chart is symmetric around 1, and compare decimal numbers. In comparing decimal numbers, students are required to consider the units involved. Students communicate their knowledge of decimals through discussion and then apply their learning to add decimals, recognizing the need to convert to like units when necessary.

MP.8   **Look for and express regularity in repeated reasoning.** As they progress through this module, students have multiple opportunities to explore the relationships between and among units of ones, tenths, and hundredths. Relationships between adjacent place values, for example, are the same on the right side of the decimal point as they are on the left side, and students investigate this fact working with tenths and hundredths. Further, adding tenths and hundredths requires finding like units just as it does with whole numbers, such as when adding centimeters and meters. Students come to understand equivalence, conversions, comparisons, and addition involving decimal fractions.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
4.NF.6 4.NBT.1 4.MD.1	A	**Exploration of Tenths**	3
		Lesson 1: Use metric measurement to model the decomposition of one whole into tenths.	
		Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.	
		Lesson 3: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.	
4.NF.5 4.NF.6 4.NBT.1 4.NF.1 4.NF.7 4.MD.1	B	**Tenths and Hundredths**	5
		Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.	
		Lesson 5: Model the equivalence of tenths and hundredths using the area model and place value disks.	
		Lesson 6: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.	
		Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.	
		Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.	
		Mid-Module Assessment: Topics A–B (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
4.NF.7 4.MD.1 4.MD.2	C	**Decimal Comparison**	3
		Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions.	
		Lesson 10: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.	
		Lesson 11: Compare and order mixed numbers in various forms.	

Standards		Topics and Objectives	Days
4.NF.5 4.NF.6 4.NF.3c 4.MD.1	D	**Addition with Tenths and Hundredths** Lesson 12: Apply understanding of fraction equivalence to add tenths and hundredths. Lesson 13: Add decimal numbers by converting to fraction form. Lesson 14: Solve word problems involving the addition of measurements in decimal form.	3
4.MD.2 4.NF.5 4.NF.6	E	**Money Amounts as Decimal Numbers** Lesson 15: Express money amounts given in various forms as decimal numbers. Lesson 16: Solve word problems involving money.	2
		End-of-Module Assessment: Topics A–E (assessment 1 day, return ½ day, remediation or further applications ½ day)	2
Total Number of Instructional Days			**20**

Terminology

New or Recently Introduced Terms

- Decimal expanded form (e.g., $(2 \times 10) + (4 \times 1) + (5 \times 0.1) + (9 \times 0.01) = 24.59$)
- Decimal fraction (a fraction with a denominator of 10, 100, 1,000, etc.)
- Decimal number (a number written using place value units that are powers of 10)
- Decimal point (a period used to separate the whole number part from the fractional part of a decimal number)
- Fraction expanded form (e.g., $(2 \times 10) + (4 \times 1) + (5 \times \frac{1}{10}) + (9 \times \frac{1}{100}) = 24\frac{59}{100}$)
- Hundredth (a place value unit such that 100 hundredths equals 1 one)
- Tenth (a place value unit such that 10 tenths equals 1 one)

Familiar Terms and Symbols [2]

- Expanded form (e.g., $100 + 30 + 5 = 135$)
- Fraction (a numerical quantity that is not a whole number, e.g., $\frac{1}{3}$)

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[2] These are terms and symbols students have seen previously.

EUREKA MATH™   **Module 6:**   Decimal Fractions   engage^ny

Suggested Tools and Representations

- 1-liter container with milliliter marks
- Area model
- Centimeter ruler
- Decimal place value disks (tenths and hundredths)
- Digital scale
- Meter stick
- Number line
- Place value chart with decimals to hundredths
- Tape diagram
- Whole number place value disks (hundreds, tens, and ones)

Scaffolds[3]

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units,* please refer to "How to Implement *A Story of Units*."

Assessment Summary

Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic B	Constructed response with rubric	4.NF.5 4.NF.6
End-of-Module Assessment Task	After Topic E	Constructed response with rubric	4.NF.5 4.NF.6 4.NF.7 4.MD.2

[3]Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

4
GRADE

Mathematics Curriculum

Topic A
Exploration of Tenths

4.NF.6, 4.NBT.1, 4.MD.1

Focus Standard:	4.NF.6	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
Instructional Days:	3	
Coherence -Links from:	G3–M2	Place Value and Problem Solving with Units of Measure
	G3–M5	Fractions as Numbers on the Number Line
-Links to:	G5–M1	Place Value and Decimal Fractions

In Topic A, students use their understanding of fractions to explore tenths. In Lesson 1, students use metric measurement and see tenths in relation to one whole in the context of 1 kilogram, 1 meter, and 1 centimeter. Using bags of rice, each weighing $\frac{1}{10}$ kilogram, students see that the weight of 10 bags is equal to 1 kilogram. Through further exploration and observation of a digital scale, students learn that $\frac{1}{10}$ kilogram can also be expressed as 0.1 kilogram, that $\frac{2}{10}$ kilogram can be expressed as 0.2 kilogram, and that all expressions of tenths in fraction form (up to one whole) can be expressed in decimal form as well. Students then use their knowledge of pairs of 10 to determine how many more tenths are needed to bring a given number of tenths up to one whole. To bring together this metric measurement experience through a more abstract representation, tenths are represented on the number line and with tape diagrams as pictured below. Students express tenths as decimal fractions, are introduced to decimal notation, and write statements of equivalence in unit, fraction, and decimal forms (e.g., 3 tenths = $\frac{3}{10}$ = 0.3) (**4.NF.6**). Finally, meters and centimeters are decomposed into 10 equal parts in a manner similar to that in which 1 kilogram was decomposed.
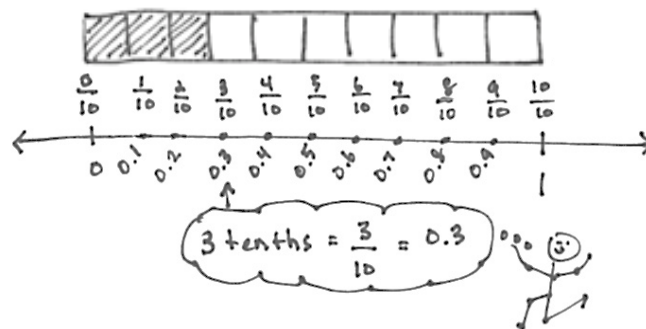
In Lesson 2, students return to the use of metric measurement, this time to investigate decimal fractions greater than 1. They use a centimeter ruler to draw lines that measure, for example, $2\frac{4}{10}$ or $6\frac{8}{10}$ centimeters, and recognize that those numbers can also be expressed in unit form as 24 tenths centimeters or 68 tenths centimeters. Students represent decimal numbers using the area model and see that numbers containing ones and fractions (i.e., mixed numbers) can also be expressed using decimal notation (e.g., 2.4 or 6.8); they also write more sophisticated statements of equivalence (e.g., $2\frac{4}{10} = 2 + \frac{4}{10}$ and 2.4 = 2 + 0.4) (**4.NF.6**).

2 ones   4 tenths
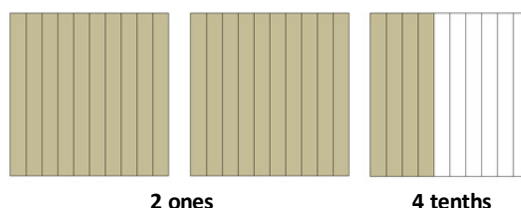
In Lesson 3, students work with place value disks and the number line to represent and identify decimal numbers with tenths as a unit. To explore the place value of each unit in a decimal number with tenths, students use place value disks to rename groups of 10 tenths as ones. Next, students learn to record the value of each digit of a mixed number in fraction expanded form, followed by decimal expanded form (e.g., 2 ones 4 tenths = $2\frac{4}{10}$ = $(2 \times 1) + (4 \times \frac{1}{10})$ and 2.4 = $(2 \times 1) + (4 \times 0.1)$). Finally, students model the value of decimal fractions within a mixed number by plotting decimal numbers on the number line.
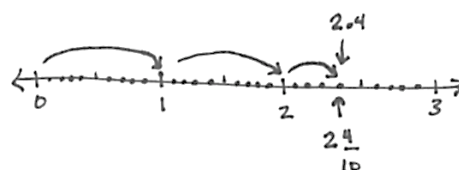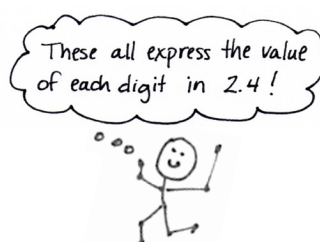
A Teaching Sequence Toward Mastery of Exploration of Tenths
Objective 1: Use metric measurement to model the decomposition of one whole into tenths. (Lesson 1)
Objective 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers. (Lesson 2)
Objective 3: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form. (Lesson 3)

Lesson 1

Objective: Use metric measurement to model the decomposition of one whole into tenths.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
□ Concept Development   (38 minutes)
■ Student Debrief   (10 minutes)
   Total Time   **(60 minutes)**
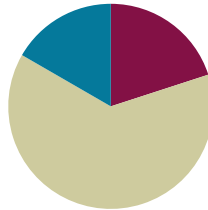
Fluency Practice (12 minutes)

▪ Divide by 10 **3.NBT.3**   (4 minutes)
▪ Sprint: Divide by 10 **3.NBT.3**   (8 minutes)

Divide by 10 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for today's lesson.

   T: (Project a tape diagram with a value of 20 partitioned into 10 units.) Say the whole.
   S: 20.
   T: How many units is 20 divided into?
   S: 10.
   T: Say the division sentence.
   S: 20 ÷ 10 = 2.
   T: (Write 2 inside each unit. Write 20 ÷ 10 = 2 beneath the diagram.)

20

| 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 |

$20 \div 10 = 2$
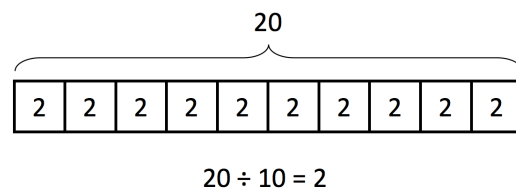
Continue with the following possible sequence: 200 ÷ 10, 240 ÷ 10, 400 ÷ 10, 430 ÷ 10, 850 ÷ 10, 8,500 ÷ 10, 8,570 ÷ 10, and 6,280 ÷ 10.

Sprint: Divide by 10 (8 minutes)

Materials: (S) Divide by 10 Sprint

Note: This Sprint prepares students for today's lesson.

Concept Development (38 minutes)

Materials: (T) 10 0.1-kilogram bags of rice, digital scale, 1-meter strip of paper, sticky notes, meter stick
(S) Meter stick (per pair), blank meter strip of paper, centimeter ruler, markers or crayons, blank paper

Note: In preparing this lesson's materials, consider the following. If a digital scale is not available, a pan balance can be used with 100-gram weights labeled as 0.1 kg. Cash register tape can be used to make meter strip papers. During Activity 2, use sticky notes to label each of the 10 1-meter strips of paper with one number: 0.1 m, 0.2 m, 0.3 m, …, 1.0 m.

Activity 1: Compose and decompose 1 kilogram, representing tenths in fraction form and decimal form.

T: (Place 10 bags of rice on the scale.) Here are 10 equal bags of rice. Together, all of this rice weighs 1 kilogram.

T: Let's draw a tape diagram to show the total amount of rice. Draw the tape as long as you can on your blank paper. What is our total amount?

S: 1 kilogram.

T: Let's write 1 kg above the tape diagram to show that the whole tape represents 1 kilogram.

T: How can we represent the 10 equal bags on the tape diagram?

S: Make 10 equal parts.

T: Partition your tape diagram to show 10 equal parts. Each of these parts represents what fraction of the whole?

S: 1 tenth! (Divide the tape diagram into 10 equal parts.)

T: (Remove all bags from the scale. Hold 1 bag in front of the class.) What fractional part of 1 kilogram is 1 bag? Point to the part this 1 bag represents on your tape diagram.

S: $\frac{1}{10}$. (Point to 1 part.)

T: Let's write the weight of this bag on your tape diagram. What is the weight of 1 bag?

S: $\frac{1}{10}$ kilogram.

T/S: (Write $\frac{1}{10}$ kg.)

T: (Place the second bag of rice in front of the class.) What is the weight of 2 bags?

S: $\frac{2}{10}$ kilogram.
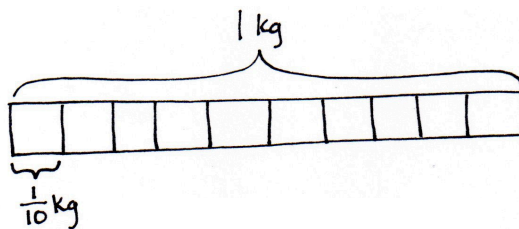
Continue to count by tenths to compose 1 kilogram.

T: Let's make a number line the same length as the tape diagram, and mark the tenths to match the parts of the tape diagram. Label the endpoints 0 and 1.

T: Let's see what $\frac{1}{10}$ kilogram looks like on the scale. (Place 1 bag on the scale.) It says zero point one kilogram.

Lesson 1:   Use metric measurement to model the decomposition of one whole into tenths.

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T: (Write 0.1 on the number line.) This is a **decimal number**. We read this decimal as 1 **tenth**, just like the fraction $\frac{1}{10}$. The decimal form is written as zero point one. The dot in a decimal number is called a **decimal point**. (Write 1 tenth = $\frac{1}{10}$ = 0.1.) 1 tenth is written in unit form, as a **decimal fraction**, and as a decimal number. They are all equal.

T: Write 1 tenth in decimal form on your number line, just like I did.

S: (Write 0.1 on the number line.)

T: Let's see how the number in decimal form changes as we add more bags or tenths of a kilogram.

T: We can express the weight of 1 bag two ways: zero point one kilogram, or 1 tenth kilogram. Tell me the weight of 2 bags using both ways. Start with the decimal point way.

S: Zero point two kilogram. 2 tenths kilogram.

T: (Invite a few students to the front of the room. Distribute two to three bags to each student.) As we add each bag, count and see how the scale shows the weight in decimal form, and record it on your number line.

S/T: Zero point two kilogram, 2 tenths kilogram, zero point three kilogram, 3 tenths kilogram, …, zero point nine kilogram, 9 tenths kilogram, one point zero kilogram, 1 kilogram!

T: Notice the scale uses decimal form for 10 tenths. 10 tenths is equal to how many ones and how many tenths?

S: 1 one and 0 tenths.

T: So, we record that as 1 point 0. Revise your number line.

T: (Take off 2 bags to show 0.8 kg.) How many tenths are on the scale now?

S: 8 tenths kilogram.

T: Record the weight of 8 bags in fraction form and decimal form. Use an equal sign.

S: (Write $\frac{8}{10}$ kg = 0.8 kg.)

T: I have 2 bags in my hand. Write the weight of this amount of rice in fraction form and decimal form. Use an equal sign.

S: (Write $\frac{2}{10}$ kg = 0.2 kg.)

T: When I put together $\frac{2}{10}$ kilogram and $\frac{8}{10}$ kilogram, I have…?

S: 1 kilogram!

T: (Write 0.2 kilogram + 0.8 kilogram = 1 kilogram.) What other pairs of tenths would make 1 kilogram when put together?

S: $\frac{3}{10}$ kilogram and $\frac{7}{10}$ kilogram. → $\frac{6}{10}$ kilogram and $\frac{4}{10}$ kilogram.

As students share out pairs, write the number sentences using decimal form.
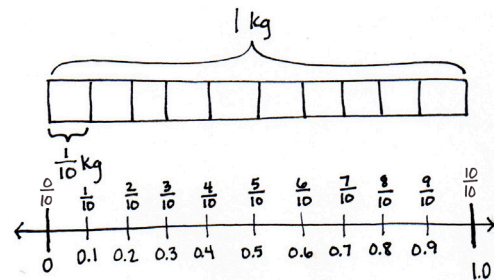
> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> Students who are not invited to place weights on the scale may enjoy shading units or placing counters in the tape diagram for each bag placed on the scale.

Activity 2: Decompose 1 meter, representing tenths in fraction form and decimal form.

Give each pair of students a meter stick and two strips of paper that are each 1 meter long. Ask them to use their meter sticks to divide each paper strip into 10 equal parts. Have them then shade with markers or crayons to show different numbers of tenths. As they work, collect strips to make an ordered set on the board, starting with 1 meter to show 10 tenths, 9 tenths, etc. Generate and record the partner each strip needs to make 1 meter next to each strip (e.g., 0.9 meter + 0.1 meter = 1 meter). Have students then generate two or three equivalent number sentences showing the equality of fraction form and decimal form (e.g., $\frac{1}{10}$ meter = 0.1 meter).

Meter Stick

2 Examples of Shaded Paper Strips:

4 tenths shaded
0.4 meter + 0.6 meter = 1 meter

9 tenths shaded
0.9 meter + 0.1 meter = 1 meter
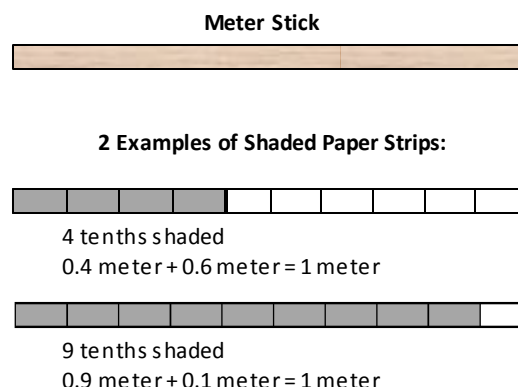
Activity 3: Decompose 1 centimeter, representing tenths in fraction form and decimal form.

T: Now that we have practiced decomposing a meter into tenths, let's use that same thinking to decompose a centimeter into tenths.

T: Take out your centimeter ruler, and draw a 1-centimeter line on the blank paper.

S: (Draw.)

T: Each centimeter has been partitioned into equal parts. How many equal parts are there from 0 to 1 centimeter?

S: 10 parts.

T: What fraction of a centimeter is one part?

S: 1 tenth.

T: How many units of 1 tenth equal 1 centimeter?

S: 10 tenths.

T: Label your line. 1 cm = $\frac{10}{10}$ cm.

T: Below your line, make a line that measures $\frac{9}{10}$ centimeter. Label your line in fraction form and decimal form.

S: (Draw a line 0.9 cm in length. Write $\frac{9}{10}$ cm = 0.9 cm.)

T: How many more tenths of a centimeter do we need to have 1 centimeter?

S: We would need 0.1 cm more.

T: (Write $\frac{9}{10}$ cm + $\frac{1}{10}$ cm = 1 cm and 0.9 cm + 0.1 cm = 1 cm.)

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> Students with low vision or other perceptual challenges may find drawing a 1-centimeter line and deciphering millimeters difficult. A centimeter stencil that students can easily trace may be beneficial. In addition to having students interact with a to-scale centimeter (such as a cube), it may help to project teacher modeling with an overhead projector or document camera, if available.

(handwritten)
— 1cm = $\frac{10}{10}$ cm
— $\frac{9}{10}$ cm = 0.9 cm $\frac{9}{10}$cm + $\frac{1}{10}$cm = 1cm   0.9cm + 0.1cm = 1cm
— $\frac{8}{10}$ cm = 0.8cm $\frac{8}{10}$cm + $\frac{2}{10}$cm = 1cm   0.8cm + 0.2cm = 1cm

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T: Now, draw a line below these lines that measures $\frac{8}{10}$ centimeter. Label this new line in fraction and decimal form. Write an addition sentence in both fraction and decimal form to show how many more tenths of a centimeter you need to get to 1 centimeter.

S: (Draw and label $\frac{8}{10}$ cm and 0.8 cm. Write $\frac{8}{10}$ cm + $\frac{2}{10}$ cm = 1 cm and 0.8 cm + 0.2 cm = 1 cm.)

T: Continue writing more pairs as you work, making a line that is $\frac{1}{10}$ centimeter shorter each time.

Select students to share so that the fraction form and decimal form of the number sentence are presented to the class.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. Some problems do not specify a method for solving. This is an intentional reduction of scaffolding that invokes MP.5, Use Appropriate Tools Strategically. Students should solve these problems using the RDW approach used for Application Problems.

For some classes, it may be appropriate to modify the assignment by specifying which problems students should work on first. With this option, let the purposeful sequencing of the Problem Set guide the selections so that problems continue to be scaffolded. Balance word problems with other problem types to ensure a range of practice. Consider assigning incomplete problems for homework or at another time during the day.

Student Debrief (10 minutes)

Lesson Objective: Use metric measurement to model the decomposition of one whole into tenths.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 2, 8 tenths liter was represented. How is that different from the 8 tenths kilogram in Problem 3? How is representing 8 tenths liter similar to representing 8 tenths kilogram?

- In Problem 2, we measured liters of water. What other type of material might we be measuring when we measure 6 tenths of a liter? Where have you seen or used liters in your everyday life?

- Look at Problem 5. How is getting to 1 centimeter similar to getting to 10, as you did in earlier grades? How did getting to 10 help you in the past? How do you think getting to 1 might help you now?

- What relationship does 1 **tenth** have to 1?

- How did your work with **decimal fractions** like $\frac{3}{10}$, $\frac{7}{10}$, or $\frac{9}{10}$ prepare you for this lesson?

- Today, we studied **decimal numbers**, and we wrote them in fraction form and decimal form. How are the two forms alike? How are they different?

- What purpose does a **decimal point** serve?

- During Fluency Practice, you divided numbers by 10. How did today's work of dividing one whole into parts relate to your fluency work? When you divide 20 by 10, what is your equal unit? When you divide 1 into 10 equal parts, what is your equal unit?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Lesson 1: Use metric measurement to model the decomposition of one whole into tenths.

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A

Number Correct: _____

Divide by 10

1.	20 ÷ 10 =		23.	50 ÷ 10 =	
2.	30 ÷ 10 =		24.	850 ÷ 10 =	
3.	40 ÷ 10 =		25.	1,850 ÷ 10 =	
4.	80 ÷ 10 =		26.	70 ÷ 10 =	
5.	50 ÷ 10 =		27.	270 ÷ 10 =	
6.	90 ÷ 10 =		28.	4,270 ÷ 10 =	
7.	70 ÷ 10 =		29.	90 ÷ 10 =	
8.	60 ÷ 10 =		30.	590 ÷ 10 =	
9.	10 ÷ 10 =		31.	7,590 ÷ 10 =	
10.	100 ÷ 10 =		32.	120 ÷ 10 =	
11.	20 ÷ 10 =		33.	1,200 ÷ 10 =	
12.	120 ÷ 10 =		34.	2,000 ÷ 10 =	
13.	50 ÷ 10 =		35.	240 ÷ 10 =	
14.	150 ÷ 10 =		36.	2,400 ÷ 10 =	
15.	80 ÷ 10 =		37.	4,000 ÷ 10 =	
16.	180 ÷ 10 =		38.	690 ÷ 10 =	
17.	280 ÷ 10 =		39.	6,900 ÷ 10 =	
18.	380 ÷ 10 =		40.	9,000 ÷ 10 =	
19.	680 ÷ 10 =		41.	940 ÷ 10 =	
20.	640 ÷ 10 =		42.	5,280 ÷ 10 =	
21.	870 ÷ 10 =		43.	6,700 ÷ 10 =	
22.	430 ÷ 10 =		44.	7,000 ÷ 10 =	

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Lesson 1: Use metric measurement to model the decomposition of one whole into tenths.

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B

Divide by 10

1.	10 ÷ 10 =	
2.	20 ÷ 10 =	
3.	30 ÷ 10 =	
4.	70 ÷ 10 =	
5.	40 ÷ 10 =	
6.	80 ÷ 10 =	
7.	60 ÷ 10 =	
8.	50 ÷ 10 =	
9.	90 ÷ 10 =	
10.	100 ÷ 10 =	
11.	30 ÷ 10 =	
12.	130 ÷ 10 =	
13.	60 ÷ 10 =	
14.	160 ÷ 10 =	
15.	90 ÷ 10 =	
16.	190 ÷ 10 =	
17.	290 ÷ 10 =	
18.	390 ÷ 10 =	
19.	690 ÷ 10 =	
20.	650 ÷ 10 =	
21.	860 ÷ 10 =	
22.	420 ÷ 10 =	

23.	40 ÷ 10 =	
24.	840 ÷ 10 =	
25.	1,840 ÷ 10 =	
26.	80 ÷ 10 =	
27.	280 ÷ 10 =	
28.	4,280 ÷ 10 =	
29.	60 ÷ 10 =	
30.	560 ÷ 10 =	
31.	7,560 ÷ 10 =	
32.	130 ÷ 10 =	
33.	1,300 ÷ 10 =	
34.	3,000 ÷ 10 =	
35.	250 ÷ 10 =	
36.	2,500 ÷ 10 =	
37.	5,000 ÷ 10 =	
38.	740 ÷ 10 =	
39.	7,400 ÷ 10 =	
40.	4,000 ÷ 10 =	
41.	910 ÷ 10 =	
42.	5,820 ÷ 10 =	
43.	7,600 ÷ 10 =	
44.	6,000 ÷ 10 =	

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Name _____   Date _____

1. Shade the first 7 units of the tape diagram. Count by tenths to label the number line using a fraction and a decimal for each point. Circle the decimal that represents the shaded part.

0    $\underline{0.1}$ ____   ____   ____   ____   ____   ____   ____   ____   1

$\frac{1}{10}$

2. Write the total amount of water in fraction form and decimal form. Shade the last bottle to show the correct amount.

1 L   0.5 L $\frac{\square}{\square}$ L = $\boxed{}$ L

1 L   0.5 L $\frac{\square}{\square}$ L = $\boxed{}$ L

1 L   0.5 L $\frac{\square}{\square}$ L = 0.9 L

3. Write the total weight of the food on each scale in fraction form or decimal form.

0.4 kg

___ kg

0.5kg

_____ kg $\frac{8}{10}$ kg   _____ kg

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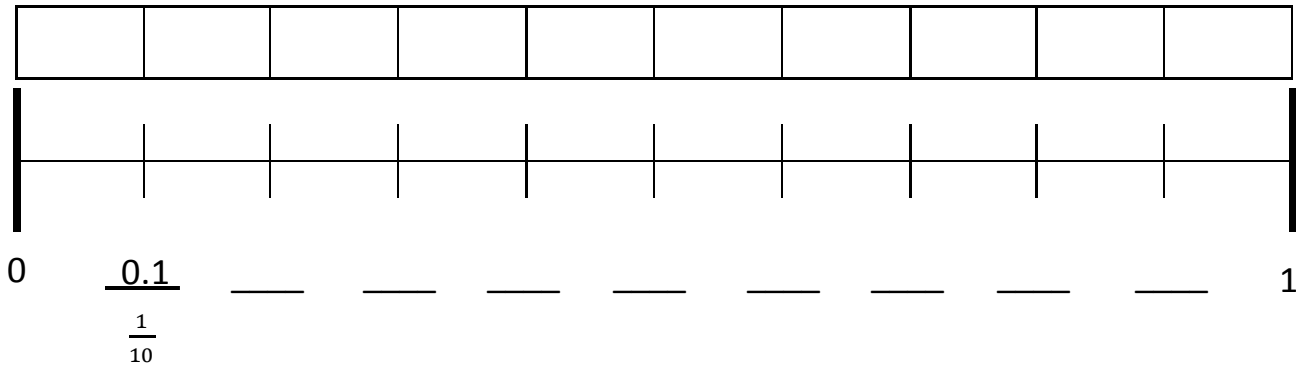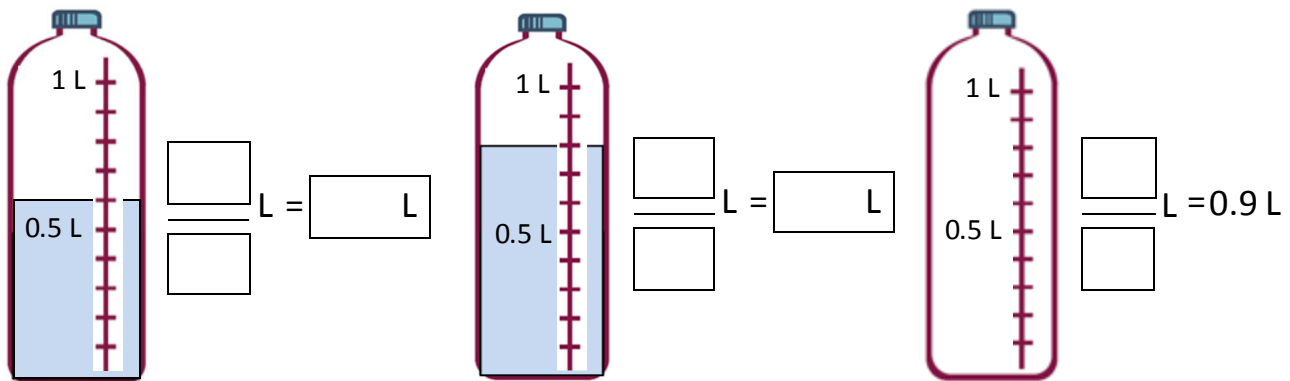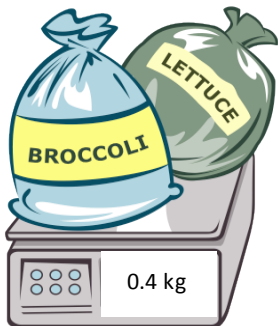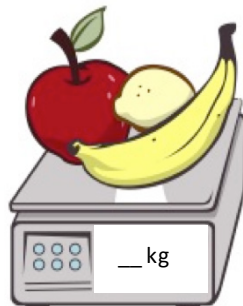
4.   Write the length of the bug in centimeters. (The drawing is not to scale.)

Fraction form: _____ cm

Decimal form: _____ cm

How far does the bug need to walk before its nose is at the 1 cm mark? _____ cm

5.   Fill in the blank to make the sentence true in both fraction form and decimal form.

a. $\frac{8}{10}$ cm + _____ cm = 1 cm   0.8 cm + _____ cm = 1.0 cm

b. $\frac{2}{10}$ cm + _____ cm = 1 cm   0.2 cm + _____ cm = 1.0 cm

c. $\frac{6}{10}$ cm + _____ cm = 1 cm   0.6 cm + _____ cm = 1.0 cm

6.   Match each amount expressed in unit form to its equivalent fraction and decimal forms.

3 tenths	$\frac{5}{10}$	0.2
5 tenths	$\frac{9}{10}$	0.6
6 tenths	$\frac{2}{10}$	0.3
9 tenths	$\frac{3}{10}$	0.5
2 tenths	$\frac{6}{10}$	0.9

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Lesson 1:   Use metric measurement to model the decomposition of one whole into tenths.

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Name _____   Date _____

1. Fill in the blank to make the sentence true in both fraction form and decimal form.

    a. $\frac{9}{10}$ cm + _____ cm = 1 cm   0.9 cm + _____ cm = 1.0 cm

    b. $\frac{4}{10}$ cm + _____ cm = 1 cm   0.4 cm + _____ cm = 1.0 cm

2. Match each amount expressed in unit form to its fraction form and decimal form.

3 tenths	$\frac{5}{10}$	0.8
8 tenths	$\frac{8}{10}$	0.3
5 tenths	$\frac{3}{10}$	0.5

Name _____   Date _____

Shade the first 4 units of the tape diagram. Count by tenths to label the number line using a fraction and a decimal for each point. Circle the decimal that represents the shaded part.

0   0.1   ____   ____   ____   ____   ____   ____   ____   ____   1

$\frac{1}{10}$

2. Write the total amount of water in fraction form and decimal form. Shade the last bottle to show the correct amount.

$\frac{\square}{\square}$ L = [___] L $\frac{\square}{\square}$ L = [___] L $\frac{\square}{\square}$ L = 0.3 L

3. Write the total weight of the food on each scale in fraction form or decimal form.

0.7 kg   [___] kg   ____ kg $\frac{6}{10}$ kg

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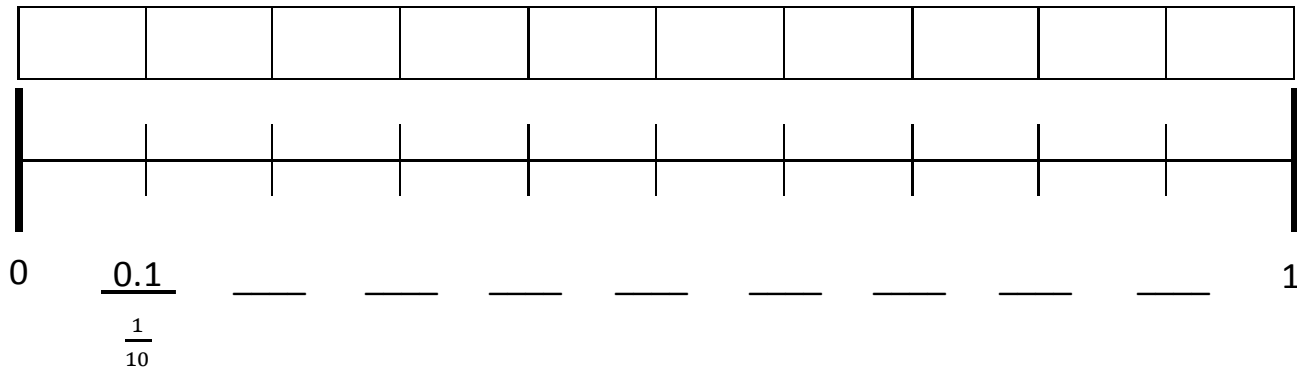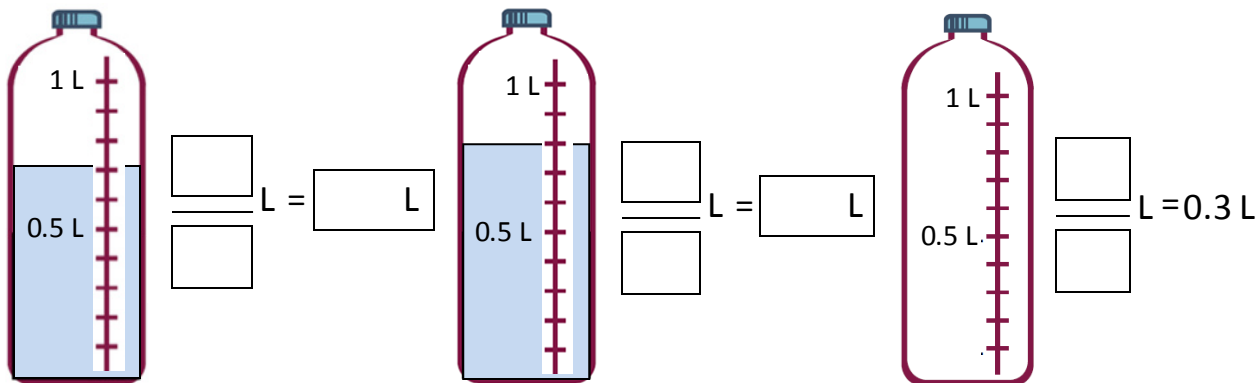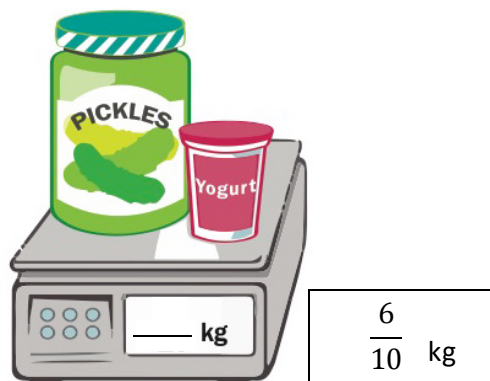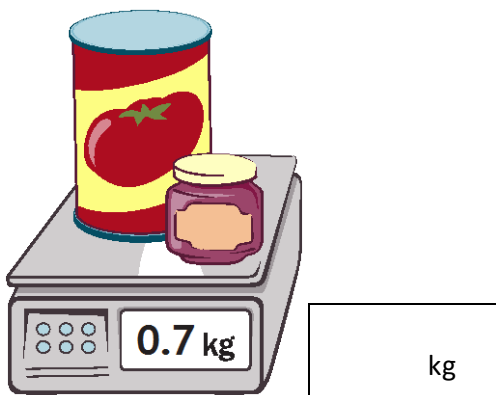
4. Write the length of the bug in centimeters. (The drawing is not to scale.)

Fraction form: _____ cm

Decimal form: _____ cm

If the bug walks 0.5 cm farther, where will its nose be? _____ cm

5. Fill in the blank to make the sentence true in both fraction and decimal form.

a. $\frac{4}{10}$ cm + _____ cm = 1 cm

0.4 cm + _____ cm = 1.0 cm

b. $\frac{3}{10}$ cm + _____ cm = 1 cm

0.3 cm + _____ cm = 1.0 cm

c. $\frac{8}{10}$ cm + _____ cm = 1 cm

0.8 cm + _____ cm = 1.0 cm

6. Match each amount expressed in unit form to its equivalent fraction and decimal.

2 tenths	$\frac{4}{10}$	0.4
4 tenths	$\frac{7}{10}$	0.6
6 tenths	$\frac{5}{10}$	0.2
7 tenths	$\frac{2}{10}$	0.5
5 tenths	$\frac{6}{10}$	0.7

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Lesson 1: Use metric measurement to model the decomposition of one whole into tenths.

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Lesson 2

Objective: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
■ Application Problem   (4 minutes)
■ Concept Development   (34 minutes)
■ Student Debrief   (10 minutes)
   Total Time   **(60 minutes)**
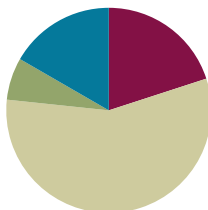
Fluency Practice (12 minutes)

▪ Divide by 10 **4.NF.6**   (4 minutes)
▪ Write the Decimal or Fraction **4.NF.6**   (3 minutes)
▪ Count by Tenths **4.NF.6**   (5 minutes)

Divide by 10 (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 1.

    T: (Project a tape diagram with a value of 100 partitioned into 10 units.) Say the whole.

    S: 100.

    T: How many units is 100 divided into?

    S: 10.

    T: Say the division sentence.

    S: $100 \div 10 = 10$.

    T: (Write 10 inside each unit. Write $100 \div 10 = 10$ beneath the diagram.)

    T: (Write $10 \div 10$.) Draw a tape diagram showing $10 \div 10$.

    S: (Draw a tape diagram partitioned into 10 units. Write 10 at the top. Write 1 inside each unit. Beneath the tape diagram, write $10 \div 10 = 1$.)

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Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

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Write the Decimal or Fraction (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 1.

- T: (Write $\frac{1}{10}$.) Say the fraction.
- S: 1 tenth.
- T: (Write $\frac{1}{10}$ = __.__.) Complete the number sentence.
- S: (Write $\frac{1}{10}$ = 0.1.)

Continue with the following possible sequence: $\frac{2}{10}, \frac{7}{10}$, and $\frac{9}{10}$.

- T: (Write 0.3 = —.) Complete the number sentence.
- S: (Write 0.3 = $\frac{3}{10}$.)

Continue with the following possible sequence: 0.4, 0.8, and 0.6.

- T: (Write $\frac{10}{10}$.) Say the fraction.
- S: 10 tenths.
- T: Complete the number sentence, writing 10 tenths as a whole number.
- S: (Write $\frac{10}{10}$ = 1.)

Count by Tenths (5 minutes)

Note: This fluency activity reviews Lesson 1.

- T: Count by ones to 10, starting at zero.
- S: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.
- T: Count by tenths to 10 tenths, starting at zero tenths.
- S: $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}, \frac{10}{10}$.
- T: 1 one is the same as how many tenths?
- S: 10 tenths.
- T: Let's count to 10 tenths again. This time, when you come to 1, say one.
- S: $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}, \frac{8}{10}, \frac{9}{10}$, 1.
- T: Count by tenths again. This time, stop when I raise my hand.
- S: $\frac{0}{10}, \frac{1}{10}, \frac{2}{10}, \frac{3}{10}$.
- T: (Raise hand.) Say 3 tenths using digits. For example, 1 tenth would be said as zero point one.
- S: Zero point three.

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Lesson 2:   Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

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T: Continue counting using fraction form.

S: $\frac{4}{10}, \frac{5}{10}, \frac{6}{10}, \frac{7}{10}$.

T: (Raise hand.) Say 7 tenths using digits.

S: Zero point seven.

T: Continue counting in fraction form.

S: $\frac{8}{10}, \frac{9}{10}, 1$.

Use the same process to count down to zero tenths.

T: Count by twos to 10 starting at zero.

S: 0, 2, 4, 6, 8, 10.

T: Count by 2 tenths to 10 tenths, starting at zero.

S: $\frac{0}{10}, \frac{2}{10}, \frac{4}{10}, \frac{6}{10}, \frac{8}{10}, \frac{10}{10}$.

T: Count by 2 tenths again. This time, when you come to the whole number, say it.

S: $\frac{0}{10}, \frac{2}{10}, \frac{4}{10}, \frac{6}{10}, \frac{8}{10}, 1$.

T: Count backward by 2 tenths, starting at 1.

S: $1, \frac{8}{10}, \frac{6}{10}, \frac{4}{10}, \frac{2}{10}, \frac{0}{10}$.

Application Problem (4 minutes)

Yesterday, Ben's bamboo plant grew 0.5 centimeter. Today it grew another $\frac{8}{10}$ centimeter. How many centimeters did Ben's bamboo plant grow in 2 days?
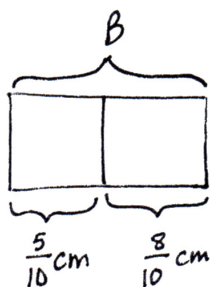
$$0.5 = \frac{5}{10}$$

$$\frac{5}{10} + \frac{8}{10} = \frac{13}{10} = 1\frac{3}{10}$$

$$B = 1\frac{3}{10} \text{ cm}$$

Ben's bamboo plant grew $1\frac{3}{10}$ centimeters in 2 days.

Note: This Application Problem builds from Module 5, in which students added fractions with like units. To do so, students use what they learned in Lesson 1 to convert a decimal number to fraction form to add.

Concept Development (34 minutes)

Materials: (T) Centimeter ruler, tenths area model (Template), document camera (S) Centimeter ruler, pencil, blank paper, tenths area model (Template), personal white board

Problem 1: Draw line segments of given lengths, and express each segment as a mixed number and a decimal.

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> Some learners may benefit from using a large print or tactile ruler that has raised lines for every centimeter. Consider adhering dried glue or rubber bands to student rulers to help learners with low vision gauge the centimeter and millimeter measures. Another possibility is providing hand-held magnifying lenses.

T: (Place a centimeter ruler under the document camera. If a document camera is unavailable, circulate to check students' work.) Using your pencil and ruler, draw a line that measures 2 centimeters. (Write 2 cm on the board.)

S: (Draw a line with the length of 2 centimeters.)

T: Extend the line 6 tenths centimeter.

S: (Extend the 2 centimeters line by 6 tenths centimeter.)

T: How many centimeters did you draw initially?

S: 2 centimeters.

T: (Label 2 cm below the line, as pictured to the right.)

T: How many tenths of a centimeter did you draw after drawing 2 centimeters?

S: 6 tenths centimeter.

T: (Label $\frac{6}{10}$ centimeter. Complete the expression 2 cm + $\frac{6}{10}$ cm below the line, as pictured to the right.)

$$2 \text{ cm} + \frac{6}{10} \text{ cm} = 2\frac{6}{10} \text{ cm}$$
$$2 \text{ cm} + 0.6 \text{ cm} = 2.6 \text{ cm}$$

T: Record a number sentence showing the total length of your line as a mixed number.

S: (Write 2 cm + $\frac{6}{10}$ cm = $2\frac{6}{10}$ cm.)

T: Let's rewrite this expression in decimal form. (Write 2 cm + 0.6 cm = 2.6 cm.) Rewrite your fraction addition in decimal form, and explain to your partner the relationship between the two number sentences and the line you drew. (Allow students time to work.)

T: $2\frac{6}{10}$ cm is written in decimal form like this: 2.6 cm. We read this as 2 and 6 tenths centimeters.

Repeat the process as necessary with $3\frac{5}{10}$ cm and $4\frac{8}{10}$ cm. Next, call out lengths verbally (e.g., 1 and 5 tenths centimeters). Students quickly draw the line and write the corresponding length in mixed number and decimal form. Suggested sequence: 1.5 cm, 5.4 cm, 3.9 cm, 9.6 cm, and 8.1 cm.

Problem 2: Use the area model to represent tenths as fractions greater than 1 and as decimal numbers.

T: (Cover up the ruler to show only 1 cm.) How many tenths are in 1?

S: 10 tenths.

MP.2

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Lesson 2:   Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

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MP.2

T: (Reveal another centimeter, showing 2 cm.) How many tenths are in 2?

S: 20 tenths.

T: (Reveal 2.6 cm.) How many tenths are in 2 and 6 tenths?

S: 26 tenths.

T: Express 26 tenths in fraction form.

S: (Write $\frac{26}{10}$.)

T: (Write $\frac{20}{10}$ cm + $\frac{6}{10}$ cm = $\frac{26}{10}$ cm.)

T: (Place the tenths area model template in a personal white board as students do the same, turn the board horizontally, and project it with a document camera.) How many rectangles are on your template?

S: 5 rectangles.

T: Each rectangle represents 1 one. How many ones do we have?

S: 5 ones.

T: Each rectangle has been partitioned equally. How many tenths are there in all?

S: 50 tenths.

T: (Write $2\frac{6}{10}$.)

T: How many ones are in this number?

S: 2 ones.

T: (Begin showing the number bond, taking out 2.) Shade in 2 ones.

S: (Shade in 2 rectangles.)

T: How many tenths do we still need to shade in?

S: 6 tenths.

T: (Complete the number bond by writing $\frac{6}{10}$.) Shade in 6 tenths more.

T: (As students are shading their template, write $2\frac{6}{10} = 2 + \frac{6}{10}$.)

T: With your partner, rewrite $2 + \frac{6}{10}$, using decimal form to add the tenths.

S: (Write 2 + 0.6.)

T: 2 + 0.6 can be written as …?

S: 2 point 6.

T: (Write 2.6 = 2 + 0.6.) With your partner, draw a number bond, this time using decimal form.
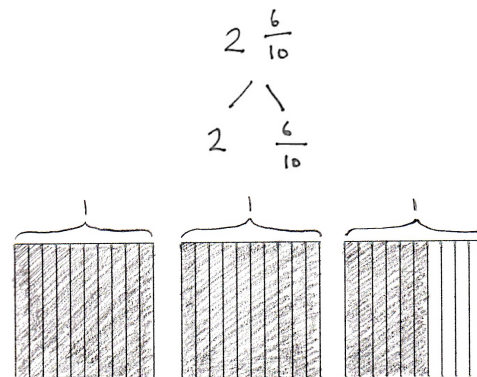
Students erase their templates. Continue the process with $2\frac{7}{10}$, $2\frac{2}{10}$, $3\frac{2}{10}$, $\frac{31}{10}$, $\frac{48}{10}$, and $\frac{26}{10}$. When appropriate, conclude each experience by asking how many more are needed to get to the next whole number, as demonstrated as follows:

Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

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T: You just shaded $3\frac{2}{10}$ and wrote this mixed number as 3 + 0.2 = 3.2. Look at your area model. How many tenths do you need to get to 4 ones?

S: 8 tenths.

T: How do you know?

S: I looked at the area model and saw that 8 tenths more have to be shaded in to complete one whole. → 2 tenths plus 8 tenths equals 10 tenths, and that makes one whole.

T: Express 8 tenths as a fraction and decimal.

With the final two or three examples, extend the question by asking how many more tenths are needed to get to 5.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Look at Problems 1(a) and 2(a). What do you notice? How could you apply what you did in Problem 2(a) to Problem 1(a)? Are there other similarities within Problems 1 and 2?

- Look at Problem 2(e). How did you know how much of the rectangles to shade in? What is the most efficient way to determine how many rectangles you would need to shade in?

Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

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- Look at Problem 2(e) with your partner. Explain to each other how you decided how much more is needed to get to 5.
- How did the Application Problem connect to today's lesson with decimal fractions?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing the students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

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Name _____   Date _____

1. For each length given below, draw a line segment to match. Express each measurement as an equivalent mixed number.

   a. 2.6 cm

   b. 3.4 cm

   c. 3.7 cm

   d. 4.2 cm

   e. 2.5 cm

2. Write the following as equivalent decimals. Then, model and rename the number as shown below.

   a. 2 ones and 6 tenths = _____
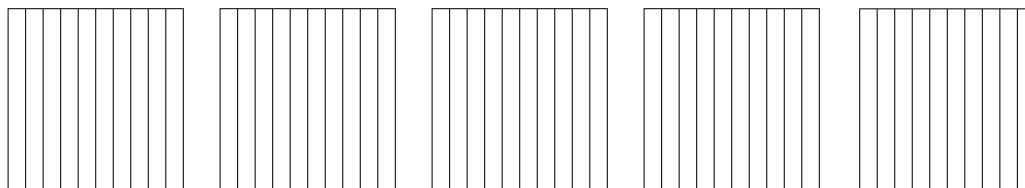
   $2\frac{6}{10} = 2 + \frac{6}{10} = 2 + 0.6 = 2.6$

b. 4 ones and 2 tenths = _____
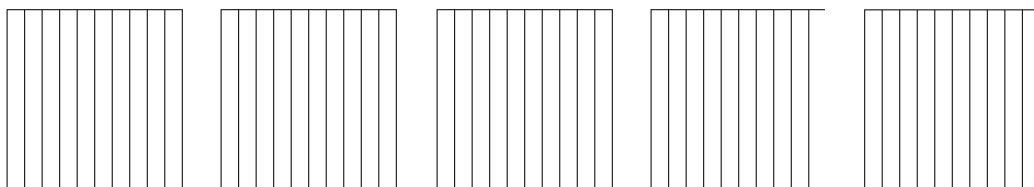
c. $3\frac{4}{10}$ = _____
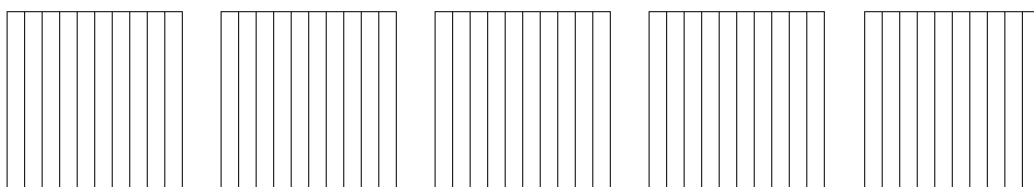
d. $2\frac{5}{10}$ = _____
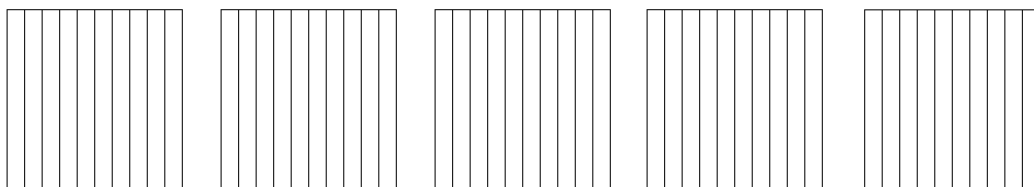
How much more is needed to get to 5? _____

e. $\frac{37}{10}$ = _____

How much more is needed to get to 5? _____

Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

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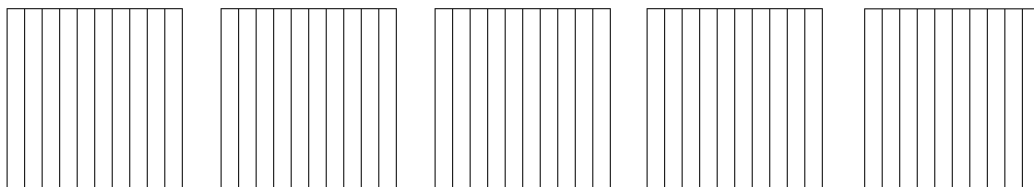
Name _____   Date _____

1. For the length given below, draw a line segment to match. Express the measurement as an equivalent mixed number.

   4.8 cm

2. Write the following in decimal form and as a mixed number. Shade the area model to match.

   a. 3 ones and 7 tenths = _____ = _____

   b. $\frac{24}{10}$ = _____ = _____

   How much more is needed to get to 5? _____

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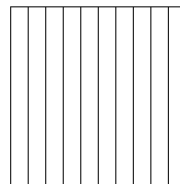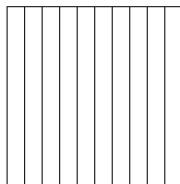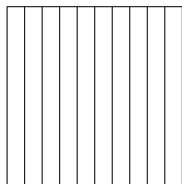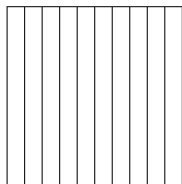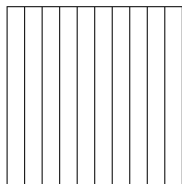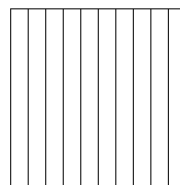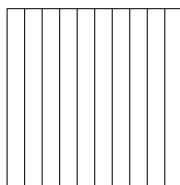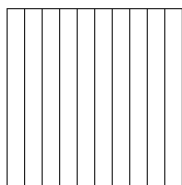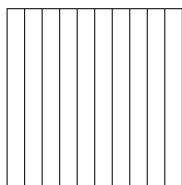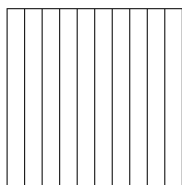
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Name _____   Date _____

1. For each length given below, draw a line segment to match. Express each measurement as an equivalent mixed number.

   a. 2.6 cm

   b. 3.5 cm

   c. 1.7 cm

   d. 4.3 cm

   e. 2.2 cm

2. Write the following in decimal form. Then, model and rename the number as shown below.

   a. 2 ones and 4 tenths = _____

   $2\frac{4}{10} = 2 + \frac{4}{10} = 2 + 0.4 = 2.4$

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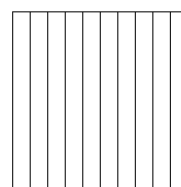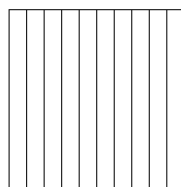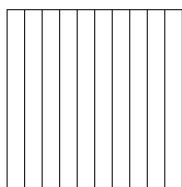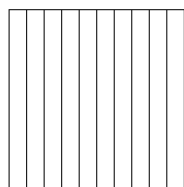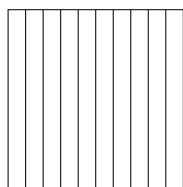
b.   3 ones and 8 tenths = _____
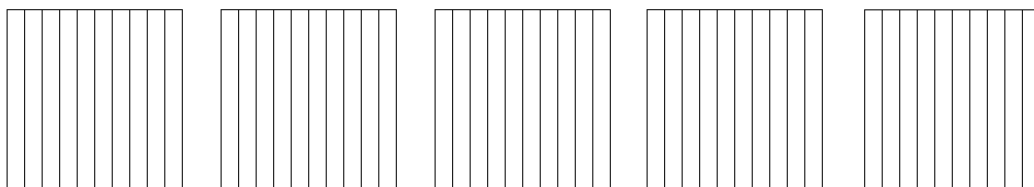
c. $4\frac{1}{10}$ = _____
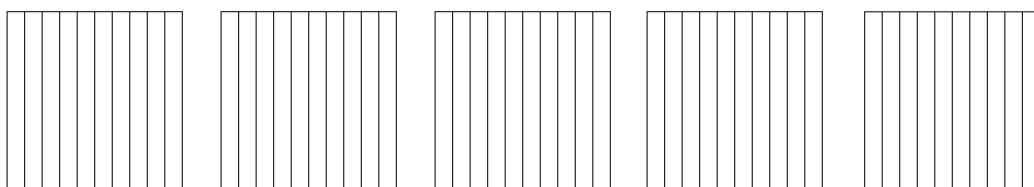
d. $1\frac{4}{10}$ = _____
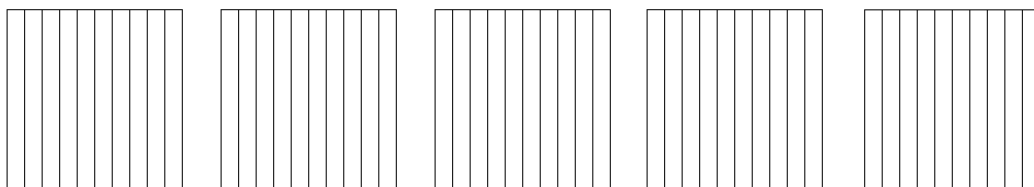
How much more is needed to get to 5? _____

e. $\frac{33}{10}$ = _____

How much more is needed to get to 5? _____

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tenths area model

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Lesson 2: Use metric measurement and area models to represent tenths as fractions greater than 1 and decimal numbers.

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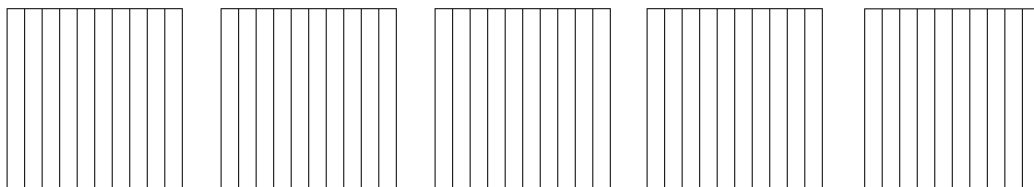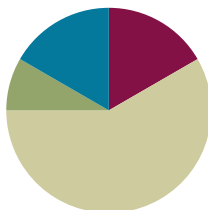
Lesson 3

Objective: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

Suggested Lesson Structure

■ Fluency Practice   (10 minutes)
■ Application Problem   (5 minutes)
■ Concept Development   (35 minutes)
■ Student Debrief   (10 minutes)
   Total Time   **(60 minutes)**

Fluency Practice (10 minutes)

▪ Write the Decimal or Fraction **4.NF.6**   (5 minutes)
▪ Count by Tenths **4.NF.6**   (5 minutes)

Write the Decimal or Fraction (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 1–2.

T: (Write $\frac{1}{10}$.) Say the fraction.

S: 1 tenth.

T: (Write $\frac{1}{10}$ = __.__.) Write 1 tenth as a decimal to complete the number sentence.

S: (Write $\frac{1}{10}$ = 0.1.)

Continue with the following possible sequence: $\frac{2}{10}$, $\frac{7}{10}$, and $\frac{9}{10}$.

T: (Write 0.3 = —.) Write zero point three as a fraction to complete the number sentence.

S: (Write 0.3 = $\frac{3}{10}$.)

Continue with the following possible sequence: 0.4, 0.8, and 0.6.

T: (Write $\frac{10}{10}$.) 10 tenths equals what whole number?

S: 1.

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T: (Write $\frac{10}{10}$ = 1. Beneath it, write $\frac{30}{10}$.) How many ones is 30 tenths?

S: 3 ones.

T: (Write $\frac{50}{10}$.) How many ones is 50 tenths?

S: 5 ones.

T: (Write $\frac{13}{10}$.) Write 13 tenths as a mixed number.

S: (Write $\frac{13}{10}$ = $1\frac{3}{10}$.)

T: (Write $\frac{13}{10}$ = $1\frac{3}{10}$ = __.__.) Write $1\frac{3}{10}$ in decimal form.

S: (Write $\frac{13}{10}$ = $1\frac{3}{10}$ = 1.3.)

Continue with the following possible sequence: $\frac{17}{10}, \frac{37}{10}, \frac{34}{10}$, and $\frac{49}{10}$.

T: (Write 2.1.) Write two point one as a mixed number.

S: (Write 2.1 = $2\frac{1}{10}$.)

Continue with the following possible sequence: 3.1, 5.1, 5.9, and 1.7.

Count by Tenths (5 minutes)

Materials: (T) Personal white board

Note: This fluency activity reviews Lessons 1–2.

T: Count by fives to 50, starting at zero.

$\frac{0}{10}$	$\frac{5}{10}$	$\frac{10}{10}$	$\frac{15}{10}$	$\frac{20}{10}$	$\frac{25}{10}$	$\frac{30}{10}$	$\frac{35}{10}$	$\frac{40}{10}$	$\frac{45}{10}$	$\frac{50}{10}$
0		1		2		3		4		5

S: 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50.

T: Count by 5 tenths to 50 tenths, starting at 0 tenths. (Write as students count.)

S: $\frac{0}{10}, \frac{5}{10}, \frac{10}{10}, \frac{15}{10}, \frac{20}{10}, \frac{25}{10}, \frac{30}{10}, \frac{35}{10}, \frac{40}{10}, \frac{45}{10}, \frac{50}{10}$.

T: 1 is the same as how many tenths?

S: 10 tenths.

T: (Beneath $\frac{10}{10}$, write 1.)

Continue the process, identifying the number of tenths in 2, 3, 4, and 5.

T: Let's count by 5 tenths again. This time, when you come to a whole number, say the whole number. Try not to look at the board.

S: $\frac{0}{10}, \frac{5}{10}$, 1, $\frac{15}{10}$, 2, $\frac{25}{10}$, 3, $\frac{35}{10}$, 4, $\frac{45}{10}$, 5.

Lesson 3:   Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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T: Count backward by 5 tenths, starting at 5.

S: 5, $\frac{45}{10}$, 4, $\frac{35}{10}$, 3, $\frac{25}{10}$, 2, $\frac{15}{10}$, 1, $\frac{5}{10}$, $\frac{0}{10}$.

T: Count by 5 tenths again. This time, stop when I raise my hand.

S: $\frac{0}{10}$, $\frac{5}{10}$, 1, $\frac{15}{10}$.

T: (Raise hand.) Say 15 tenths using digits.

S: One point five.

Continue the process counting up to 5 and down from 5, asking students to say the improper fractions using digits.

Application Problem (5 minutes)

Ed bought 4 pieces of salmon weighing a total of 2 kilograms. One piece weighed $\frac{4}{10}$ kg, and two of the pieces weighed $\frac{5}{10}$ kg each. What was the weight of the fourth piece of salmon?
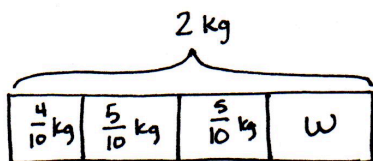
Note: This Application Problem anticipates decimal fraction addition and reinforces the concept of how many more to make one.

Concept Development (35 minutes)

Materials: (T/S) Whole number place value disks (tens and ones), decimal place value disks (tenths), personal white board, tenths on a number line (Template)

Problem 1: Make groups of 10 tenths to rename as ones. Write the number in decimal form.

T: With a partner, use place value disks to show 21 units of 1 tenth in five-group formation.

S: (Lay out 21 disks, all tenths, in five-group formation, as shown.)

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Lesson 3:   Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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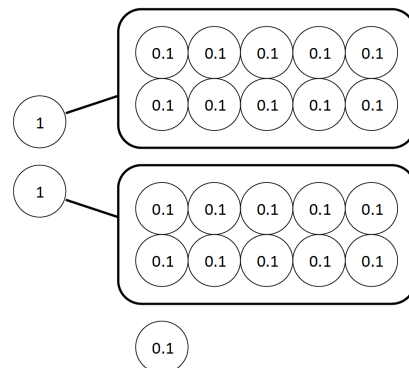
T: Talk with your partner. Is there any way we can use *fewer* disks to show this same value?

S: We can bundle 10 tenths to make one. → There are 2 groups of 10 tenths, so we can show 21 tenths as 2 ones 1 tenth. → In the five-groups, I can see 2 groups of 10 disks. 10 tenths is 1 whole. We have 1 (circling group with finger), 2 (circling group with finger) groups that make 2 ones, and then 1 tenth (touching final 0.1 disk.)

T: Let's group 10 tenths together and trade them for…?

S: 1 one.

T: How many times can we do this?

S: 1 more time. → 2 times.

T: What disks do we have now?

S: 2 ones and 1 tenth.

T: Express this number in decimal form.

S: (Write 2.1.)

T: How many more tenths would we have needed to have 3 ones?

S: 9 tenths more. → 0.9.

Repeat the process using disks to model 17 tenths. Then, continue the process having the students draw disks for 24 tenths. Have students circle the disks being bundled.

Problem 2: Represent mixed numbers with units of tens, ones, and tenths in expanded form.

T: Hold up a place value disk with a value of 1 ten. We say the value of this disk is…?

S: 1 ten. → Ten.

T: (Draw or show 4 tens disks.) The total value of 4 of these is…?

S: 4 tens. → Forty.

T: 4 tens written as a multiplication expression is?

S: 4 × 1 ten. → 4 × 10.

T: (Write the expression below the disks, as pictured to the right.) 4 × 10 is…?

MP.4

S: 40.

T: (Complete the number sentence. Draw or show 2 ones disks.) The total value of these 2 disks is…?

S: 2 ones. → Two.

T: 2 ones written as a multiplication expression is…?

S: 2 × 1.

T: (Write the expression below the disks, as pictured to the right.) (4 × 10) + (2 × 1) is…?

S: 42.

$$4 \times 10 = 40$$

$$(4 \times 10) + (2 \times 1) = 42$$

Lesson 3:   Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.
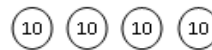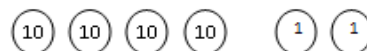
42

MP.4

T: (Complete the number sentence. Draw or show a tenth disk.) This place value disk says zero point one on it. We say the value of this disk is…?

S: 1 tenth.
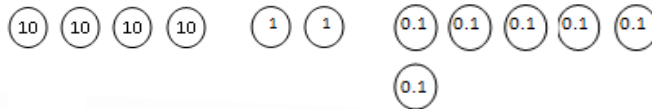
T: (Draw or show 6 one-tenth disks in five-group formation.) The total value of 6 of these disks is …?

$$(4 \times 10) + (2 \times 1) + \left(6 \times \frac{1}{10}\right) = 42 \frac{6}{10}$$

$$(4 \times 10) + (2 \times 1) + (6 \times 0.1) = 42.6$$

S: 6 tenths.

T: 6 tenths written as a multiplication expression is…?

S: $6 \times \frac{1}{10}$.

T: (Write the expression below the disks, as pictured above.) Discuss the total value of the number represented by the disks with your partner.

S: Do what is in the parentheses first, and then find the sum. $40 + 2 + \frac{6}{10}$ is $42\frac{6}{10}$. → 4 tens, 2 ones, 6 tenths. → It is like expanded form.

T: We have written $42\frac{6}{10}$ in expanded form, writing each term as a multiplication expression. Just like with whole numbers, the expanded form allows us to see the place value unit for each digit.

T: (Point to $(4 \times 10) + (2 \times 1) + (6 \times \frac{1}{10}) = 42\frac{6}{10}$.) Talk with your partner. How could you write this using **decimal expanded form** instead of **fraction expanded form**? Explain how you know.

S: (Work with partners, and write $(4 \times 10) + (2 \times 1) + (6 \times 0.1) = 42.6$.) I know that 1 tenth can be written as zero point one, and 42 and 6 tenths can be written as forty-two point six. → We looked at our disks. We had 4 tens, 2 ones, and 6 disks that had 0.1 on them. → We knew it was $42 + 0.6$, so that helped us rewrite $42\frac{6}{10}$ as 42.6.

Continue the process of showing a mixed number with place value disks, and then writing the expanded fraction form and expanded decimal form for the following numbers: 24 ones 6 tenths, 13 ones 8 tenths, and 68 ones 3 tenths. Challenge students to think how much each number needs to get to the next *one*.

Problem 3: Use the number line to model mixed numbers with units of ones and tenths.

T: (Distribute the Lesson 3 Template, tenths on a number line, and insert it into personal white boards.) Label the larger intervals from 0 to 5.

T: The segment between each whole number is divided up into how many equal parts?

S: 10 equal parts.

T: Plot a point on the number line to represent 4 and 1 tenth.

T: In the chart below your number line, let's plot the same number on a shorter number line partitioned into tenths. What will the endpoints of this shorter number line be?
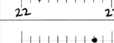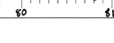
S: 4 and 5.

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Lesson 3:   Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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T: (Fill out the chart to show 4.1 plotted on a number line between 4 and 5, in decimal form, as a mixed number, and in expanded form.)

Point	Number Line	Decimal Form	Mixed Number (ones and fraction form)	Expanded Form (fraction or decimal form)	How much more is needed to get to the next one?
a.		4.1	$4\frac{1}{10}$	$(4\times1)+(1\times\frac{1}{10})=4\frac{1}{10}$ $(4\times1)+(1\times0.1)=4.1$	$\frac{9}{10}$ or 0.9
b.		32.5	$32\frac{5}{10}$	$(3\times10)+(2\times1)+(5\times\frac{1}{10})$ $=32\frac{5}{10}$ $(3\times10)+(2\times1)+(5\times0.1)$ $=32.5$	$\frac{5}{10}$ or 0.5
c.		40.7	$40\frac{7}{10}$	$(4\times10)+(7\times\frac{1}{10})=40\frac{7}{10}$ $(4\times10)+(7\times0.1)=40.7$	$\frac{3}{10}$ or 0.3
d.		90.9	$90\frac{9}{10}$	$(9\times10)+(9\times\frac{1}{10})=90\frac{9}{10}$ $(9\times10)+(9\times0.1)=90.9$	$\frac{1}{10}$ or 0.1

tenths on a number line
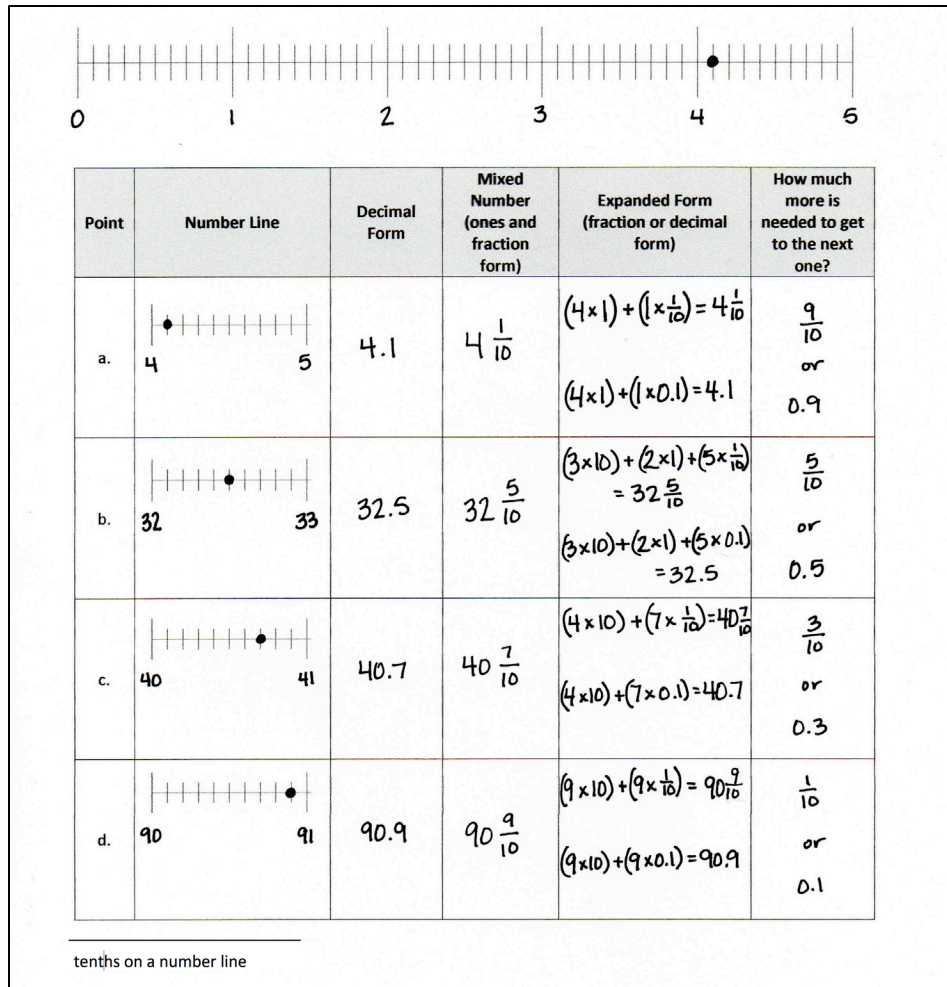
S: (Write 4 ones and 1 tenth, 4.1, $4\frac{1}{10}$, $(4\times1)+(1\times0.1)=4.1$. → $(4\times1)+(1\times\frac{1}{10})=4\frac{1}{10}$.)

T: How many more tenths are needed to get to 5? Explain to your partner how you know, and complete the final column of the chart.

S: 9 tenths. → $\frac{9}{10}$. → 0.9. → I know because it takes 10 tenths to make a one. If we have 1 tenth, we need 9 more tenths to make 1.

Repeat the process by naming the following points for students to plot. Then, have them complete and share their charts. The longer number line with 5 whole number intervals can either be relabeled to show a broader range of numbers than those included in the chart or omitted for parts (b)–(d) below.

b. 3 tens 2 ones and 5 tenths
c. 4 tens 7 tenths
d. 9 tens 9 tenths

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Lesson 3: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.
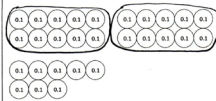
Any combination of the questions below may be used to lead the discussion.
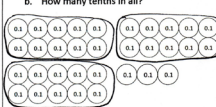
- Look at Problem 3(b). Today, we showed mixed numbers in **decimal expanded form** and **fraction expanded form**. How could you represent this number with place value disks? With an area model? Draw a line that is 17.5 cm in length.

- Look at Problem 3(a). How would you represent this number using only tenths? With your partner, use the number line or centimeter ruler to prove that 39 tenths is the same as 3 ones and 9 tenths.

- Look at Problems 2(d) and 3(c). How are these two problems alike?

- In Problems 2(c), 2(d), and 3(e) we have the same number of tens as tenths. Explain to your partner the difference in value between the tens place and the tenths place. Notice that the ones are sandwiched between the tens and tenths.
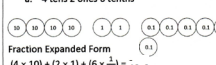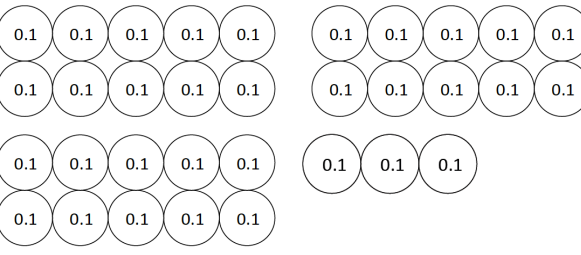
■   How did you locate points on the number line?
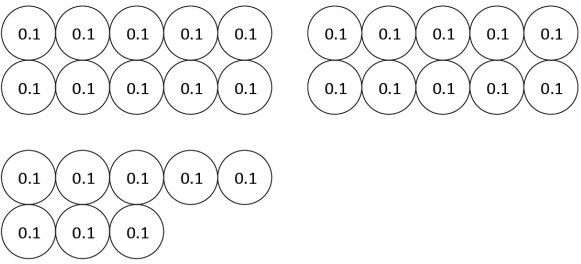
Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.
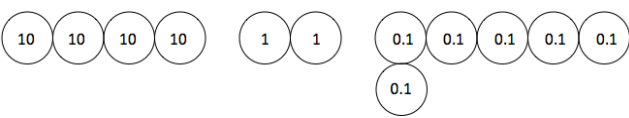
Lesson 3:   Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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Name _____ Date _____

1. Circle groups of tenths to make as many ones as possible.

a. How many tenths in all?	Write and draw the same number using ones and tenths.
There are _____ tenths.	Decimal Form: _____ How much more is needed to get to 3? _____
b. How many tenths in all?	Write and draw the same number using ones and tenths.
There are _____ tenths.	Decimal Form: _____ How much more is needed to get to 4? _____
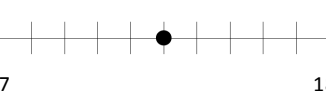
2. Draw disks to represent each number using tens, ones, and tenths. Then, show the expanded form of the number in fraction form and decimal form as shown. The first one has been completed for you.

a. 4 tens 2 ones 6 tenths	b. 1 ten 7 ones 5 tenths
Fraction Expanded Form $(4 \times 10) + (2 \times 1) + (6 \times \frac{1}{10}) = 42\frac{6}{10}$ Decimal Expanded Form $(4 \times 10) + (2 \times 1) + (6 \times 0.1) = 42.6$	

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Lesson 3: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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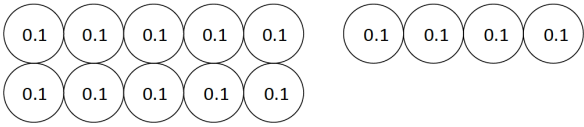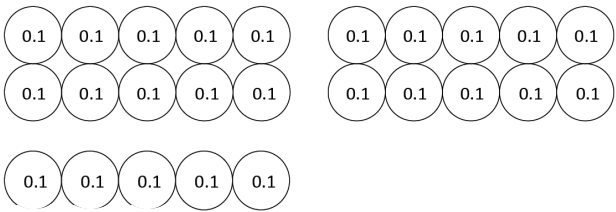| c. 2 tens 3 ones 2 tenths	d. 7 tens 4 ones 7 tenths

3. Complete the chart.
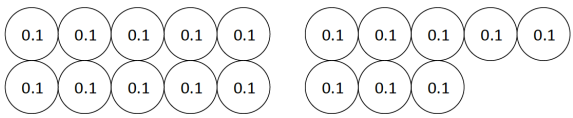
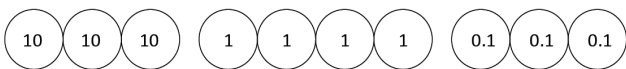| Point	Number Line	Decimal Form	Mixed Number (ones and fraction form)	Expanded Form (fraction or decimal form)	How much to get to the next one?
a.			$3\frac{9}{10}$		0.1
b.	17 18				
c.				$(7 \times 10) + (4 \times 1) + (7 \times \frac{1}{10})$	
d.			$22\frac{2}{10}$		
e.				$(8 \times 10) + (8 \times 0.1)$	

Lesson 3:   Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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Name _____   Date _____

1. Circle groups of tenths to make as many ones as possible.

How many tenths in all?	Write and draw the same number using ones and tenths.
0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 There are _____ tenths.	 Decimal Form: _____ How much more is needed to get to 2? _____

2. Complete the chart.

Point	Number Line	Decimal Form	Mixed Number (ones and fraction form)	Expanded Form (fraction or decimal form)	How much to get to the next one?
a.	├┼┼┼┼┼┼┼┼┼┤		$12\frac{9}{10}$		
b.	├┼┼┼┼┼┼┼┼┼┤	70.7			

Name _____   Date _____

1. Circle groups of tenths to make as many ones as possible.

a. How many tenths in all?	Write and draw the same number using ones and tenths.
0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 There are _____ tenths.	 Decimal Form: _____ How much more is needed to get to 2? _____
b. How many tenths in all?	Write and draw the same number using ones and tenths.
0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 0.1 There are _____ tenths.	 Decimal Form: _____ How much more is needed to get to 3? _____
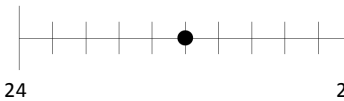
2. Draw disks to represent each number using tens, ones, and tenths. Then, show the expanded form of the number in fraction form and decimal form as shown. The first one has been completed for you.

a. 3 tens 4 ones 3 tenths	b. 5 tens 3 ones 7 tenths
10 10 10 1 1 1 1 0.1 0.1 0.1 Fraction Expanded Form $(3 \times 10) + (4 \times 1) + (3 \times \frac{1}{10}) = 34\frac{3}{10}$ Decimal Expanded Form $(3 \times 10) + (4 \times 1) + (3 \times 0.1) = 34.3$	

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c. 3 tens 2 ones 3 tenths	d. 8 tens 4 ones 8 tenths

3. Complete the chart.

Point	Number Line	Decimal Form	Mixed Number (ones and fraction form)	Expanded Form (fraction or decimal form)	How much to get to the next one?
a.			$4\frac{6}{10}$		
b.	24 25				0.5
c.				$(6 \times 10) + (3 \times 1) + (6 \times \frac{1}{10})$	
d.			$71\frac{3}{10}$		
e.				$(9 \times 10) + (9 \times 0.1)$	

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Lesson 3:   Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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Point	Number Line	Decimal Form	Mixed Number (ones and fraction form)	Expanded Form (fraction or decimal form)	How much more is needed to get to the next one?
a.					
b.					
c.					
d.					

tenths on a number line

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Lesson 3: Represent mixed numbers with units of tens, ones, and tenths with place value disks, on the number line, and in expanded form.

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Topic B
Tenths and Hundredths

4.NF.5, 4.NF.6, 4.NBT.1, 4.NF.1, 4.NF.7, 4.MD.1

Focus Standards:	4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)
	4.NF.6	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
Instructional Days:	5	
Coherence -Links from:	G3–M2	Place Value and Problem Solving with Units of Measure
	G3–M5	Fractions as Numbers on the Number Line
-Links to:	G5–M1	Place Value and Decimal Fractions

In Topic B, students decompose tenths into 10 equal parts to create hundredths. In Lesson 4, they once again use metric measurement as a basis for exploration. Using a meter stick, they locate 1 tenth meter and then locate 1 hundredth meter. They identify 1 centimeter as $\frac{1}{100}$ meter and count $\frac{1}{100}, \frac{2}{100}, \frac{3}{100}$, up to $\frac{10}{100}$, and, at the concrete level, realize the equivalence of $\frac{10}{100}$ meter and $\frac{1}{10}$ meter. They represent $\frac{1}{100}$ meter as 0.01 meter, counting up to $\frac{25}{100}$ or 0.25, both in fraction and decimal form. They then model the meter with a tape diagram and partition it into tenths, as they did in Lesson 1. Students locate 25 centimeters and see that it is equal to 25 hundredths by counting up, $\frac{10}{100}, \frac{20}{100}, \frac{21}{100}, \frac{22}{100}, \frac{23}{100}, \frac{24}{100}, \frac{25}{100}$. They represent this as $\frac{20}{100} + \frac{5}{100} = \frac{25}{100}$ and, using decimal notation, write 0.25. A number bond shows the decomposition of 0.25 into the fractional parts of $\frac{2}{10}$ and $\frac{5}{100}$.
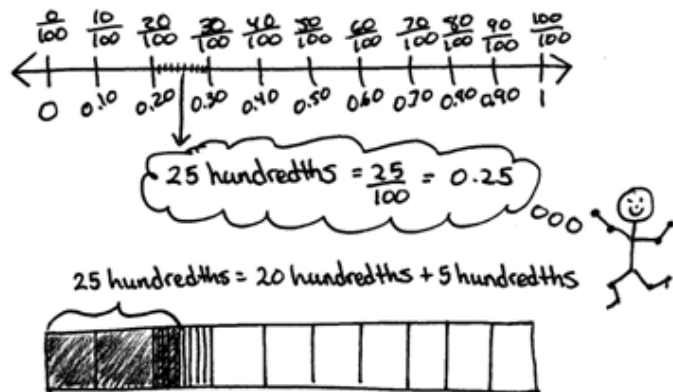
In Lesson 5, students relate hundredths to the area model (pictured below), to a tape diagram, and to place value disks. They see and represent the equivalence of tenths and hundredths pictorially and numerically.

1 hundredth $= \frac{1}{100} = 0.01$ 5 hundredths $= \frac{5}{100} = 0.05$ 25 hundredths $= \frac{25}{100} = 0.25$
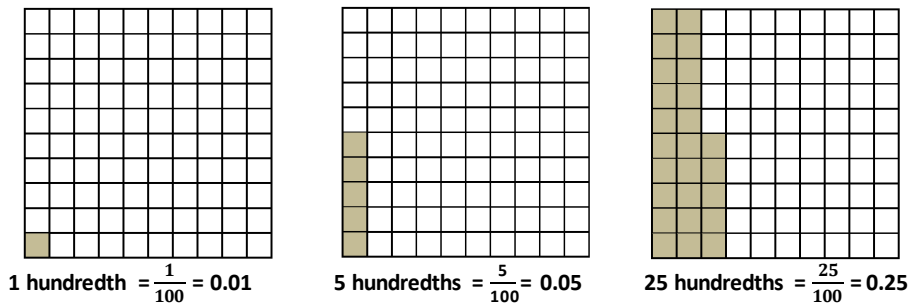
Students count up from $\frac{1}{100}$ with place value disks just as they did with centimeters in Lesson 4. This time, the 10 hundredths are traded for 1 tenth, and the equivalence is expressed as $\frac{1}{10} = \frac{10}{100} = 0.1 = 0.10$ (**4.NF.5**, **4.NF.6**). The equivalence of tenths and hundredths is also realized through multiplication and division (e.g., $\frac{1}{10} = \frac{1 \times 10}{10 \times 10} = \frac{10}{100}$ and $\frac{10}{100} = \frac{10 \div 10}{100 \div 10} = \frac{1}{10}$), establishing that 1 tenth is 10 times as much as 1 hundredth. They see, too, that 16 hundredths is 1 tenth and 6 hundredths, and that 25 hundredths is 2 tenths and 5 hundredths.
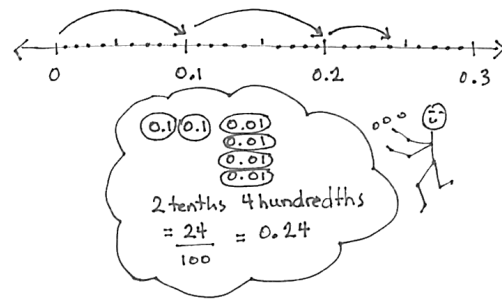
In Lesson 6, students draw representations of three-digit decimal numbers (with ones, tenths, and hundredths) with the area model.
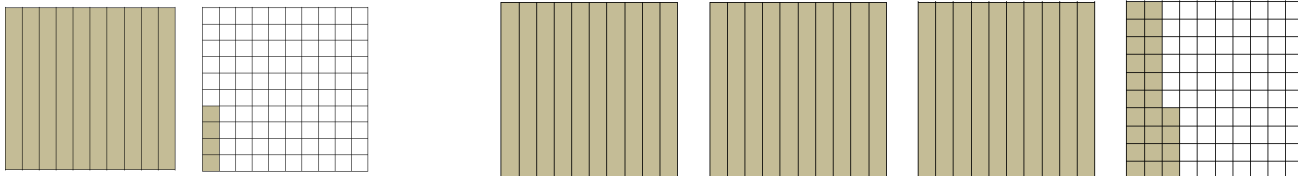
1 one 4 hundredths $= 1\frac{4}{100} = 1.04$

3 ones 24 hundredths $= 3\frac{24}{100} = 3.24$

Students also further extend their use of the number line to show the ones, tenths, and hundredths as lengths. Lesson 6 concludes with students coming to understand that tenths and hundredths each hold a special place within a decimal number, establishing that 3.80 and 3.08 are different and distinguishable values.

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Topic B: Tenths and Hundredths

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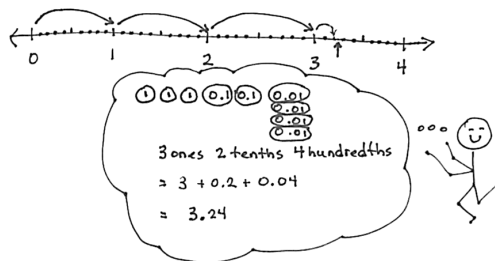
In Lesson 7, decimal numbers to hundredths are modeled with disks and written on the place value chart, where each digit's value is analyzed. The value of the total number is represented in both fraction and decimal expanded form as pictured below.
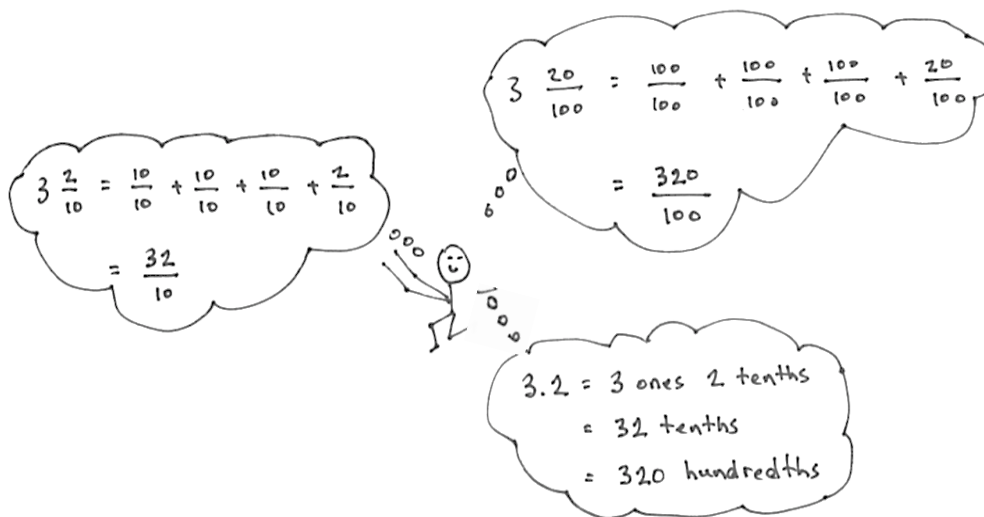
Hundreds	Tens	Ones	•	Tenths	Hundredths
3	7	8		7	3

3 hundreds 7 tens 8 ones 7 tenths 3 hundredths

$$\left(3 \times 100\right) + \left(7 \times 10\right) + \left(8 \times 1\right) + \left(7 \times \frac{1}{10}\right) + \left(3 \times \frac{1}{100}\right) = 378\frac{73}{100}$$

$$\left(3 \times 100\right) + \left(7 \times 10\right) + \left(8 \times 1\right) + \left(7 \times 0.1\right) + \left(3 \times 0.01\right) = 378.73$$

In the Debrief, students discuss the symmetry of the place value chart around 1, seeing the ones place as the "mirror" for tens and tenths and hundreds and hundredths, thereby avoiding the misconception of the *oneths* place or the decimal point itself as the point of symmetry. This understanding helps students recognize that, even as we move to the decimal side of the place value chart, a column continues to represent a unit 10 times as large as that of the column to its right.

In Lesson 8, students use what they know about fractions to represent decimal numbers in terms of different units. For example, 3.2 might be modeled as 3 ones 2 tenths, 32 tenths, or 320 hundredths. Students show these renamings in unit form, fraction form, and decimal form.

$$3\frac{2}{10} = \frac{10}{10} + \frac{10}{10} + \frac{10}{10} + \frac{2}{10}$$
$$= \frac{32}{10}$$

$$3\frac{20}{100} = \frac{100}{100} + \frac{100}{100} + \frac{100}{100} + \frac{20}{100}$$
$$= \frac{320}{100}$$

3.2 = 3 ones 2 tenths
    = 32 tenths
   = 320 hundredths

A Teaching Sequence Toward Mastery of Tenths and Hundredths

Objective 1: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.
(Lesson 4)

Objective 2: Model the equivalence of tenths and hundredths using the area model and place value disks.
(Lesson 5)

Objective 3: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.
(Lesson 6)

Objective 4: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.
(Lesson 7)

Objective 5: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.
(Lesson 8)

Lesson 4

Objective: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
■ Application Problem   (5 minutes)
■ Concept Development   (33 minutes)
■ Student Debrief   (10 minutes)
   Total Time   **(60 minutes)**
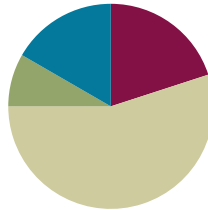
Fluency Practice (12 minutes)

▪ Sprint: Write Fractions and Decimals **4.NF.6**   (9 minutes)
▪ Count by Tenths **4.NF.6**   (3 minutes)

Sprint: Write Fractions and Decimals (9 minutes)

Materials: (S) Write Fractions and Decimals Sprint

Note: This Sprint reviews Lessons 1–3.

Count by Tenths (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lessons 1–2.

T: Count by twos to 20, starting at zero.   (Write as students count.)

$\frac{0}{10}$	$\frac{2}{10}$	$\frac{4}{10}$	$\frac{6}{10}$	$\frac{8}{10}$	$\frac{10}{10}$	$\frac{12}{10}$	$\frac{14}{10}$	$\frac{16}{10}$	$\frac{18}{10}$	$\frac{20}{10}$
0					1					2

S: 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.
T: Count by 2 tenths to 20 tenths, starting at 0 tenths. (Write as students count.)
S: $\frac{0}{10}, \frac{2}{10}, \frac{4}{10}, \frac{6}{10}, \frac{8}{10}, \frac{10}{10}, \frac{12}{10}, \frac{14}{10}, \frac{16}{10}, \frac{18}{10}, \frac{20}{10}$.

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Lesson 4:   Use meters to model the decomposition of one whole into hundredths.
                       Represent and count hundredths.

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T: 1 is the same as how many tenths?

S: 10 tenths.

T: (Beneath $\frac{10}{10}$, write 1.)

Continue the process for 2.

T: Let's count by 2 tenths again. This time, when you come to a whole number, say the whole number. Try not to look at the board.

S: $0, \frac{2}{10}, \frac{4}{10}, \frac{6}{10}, \frac{8}{10}, 1, \frac{12}{10}, \frac{14}{10}, \frac{16}{10}, \frac{18}{10}, 2$.

T: Count backward by 2 tenths, starting at 2.

S: $2, \frac{18}{10}, \frac{16}{10}, \frac{14}{10}, \frac{12}{10}, 1, \frac{8}{10}, \frac{6}{10}, \frac{4}{10}, \frac{2}{10}, \frac{0}{10}$.

Application Problem (5 minutes)

Ali is knitting a scarf that will be 2 meters long. So far, she has knitted $1\frac{2}{10}$ meters.

a. How many more meters does Ali need to knit to complete the scarf? Write the answer as a fraction and as a decimal.

b. How many more centimeters does Ali need to knit to complete the scarf?
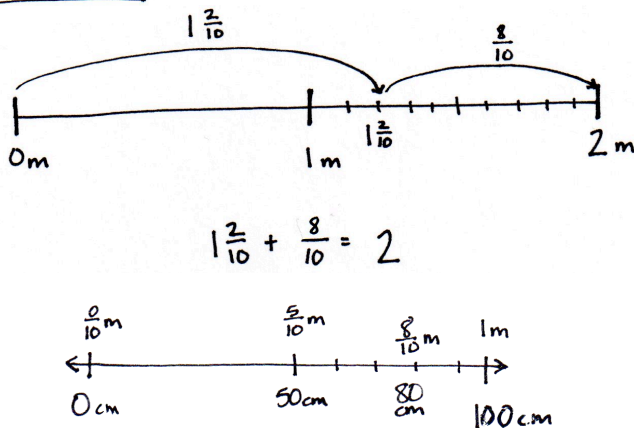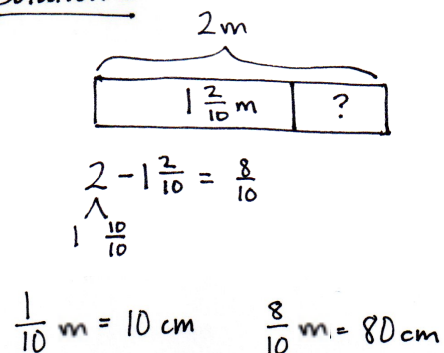
Note: This Application Problem reviews mixed decimal fractions and *counting on* to make 1 more. Revisit the problem in the Debrief to answer in hundredths meters.

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Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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Concept Development (33 minutes)

Materials: (T) Meter stick, 1-meter strip of paper partitioned into 10 equal parts by folds or dotted lines
(S) Personal white board, tape diagram in tenths (Template), pencil

Problem 1: Recognize 1 centimeter as $\frac{1}{100}$ of a meter, which can be written as $\frac{1}{100}$ m and as 0.01 m.

T: This is a meter stick. What is its length?

S: 1 meter.

T: How many centimeters are in a meter?

S: 100 centimeters.

T: (Write on the board 1 m = 100 cm.)

T: (Show centimeters on the meter stick.) A meter is made of 100 centimeters. What fraction of a meter is 1 centimeter?

S: $\frac{1}{100}$ meter.

T: (Write $\frac{1}{100}$ m = 1 cm.) In decimal form, $\frac{1}{100}$ meter can be written as zero point zero one meter. (Write 0.01 m.)

T: 1 **hundredth** is written as zero point zero one. How do you think we represent $\frac{3}{100}$ meter in decimal form? Talk with your partner, and write your thoughts on your personal white board.

S: 0.03 meter.

T: Yes. $\frac{3}{100}$ meter can be shown as a fraction or in decimal form. (Write $\frac{3}{100}$ m = 0.03 m.)

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> Be sure to enunciate *th* at the end of *hundredths* to help English language learners distinguish *hundredths* and *hundreds*. If possible, speak more slowly, pause more frequently, or couple the language with a place value chart. Check for student understanding and correct pronunciation of fraction names.

1 meter = 100 centimeters

$\frac{1}{10}$ meter = 10 centimeters

$\frac{1}{100}$ meter = 1 centimeter

1 centimeter = $\frac{1}{100}$ meter.
1 centimeter = 0.01 meter!

T: (Show the meter strip of paper.) This 1-meter paper strip is partitioned into 10 equal parts. Let's shade $\frac{1}{10}$ meter. How many centimeters equal $\frac{1}{10}$ meter?

S: 10 centimeters.

T: How many hundredths of a meter equal $\frac{1}{10}$ meter?

S: $\frac{10}{100}$ meter.

T: (Write $\frac{1}{10}$ m = $\frac{10}{100}$ m.) We can write this number as a fraction. We can also write it in decimal form. (Write 0.1 m = 0.10 m.) This (pointing to the latter) is how you express $\frac{10}{100}$ meter as a decimal.

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Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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T: Let's decompose $\frac{1}{10}$ meter into 10 smaller units to prove that this number sentence, 0.1 m = 0.10 m, is true. (Partition the tenth into 10 parts.) Is each of these new smaller units $\frac{1}{100}$ meter and 1 centimeter in length?

S: Yes.

T: Explain to your partner why.

Repeat the process by shading the next tenth of the meter. Partition it into hundredths, and have students reason about the truth of the following number sentence: $\frac{2}{10}$ m = $\frac{20}{100}$ m = 0.2 m = 0.20 m.

Problem 2: Name hundredths as tenths and some hundredths, stating the number in fraction and decimal form.

T: (Show the meter strip of paper with 2 tenths shaded.) How many tenths of this meter strip of paper are shaded?

S: $\frac{2}{10}$ meter.

T: Use the tape diagram in tenths template to represent this amount. Lightly shade 2 tenths using a pencil.

T: (Write $\frac{2}{10}$ m + $\frac{5}{100}$ m on the board.) Let's shade in $\frac{5}{100}$ meter more. What will you have to do first in order to shade $\frac{5}{100}$ meter?

S: Partition the next tenth of a meter into 10 equal parts.

S: (Partition the next tenth meter into 10 equal parts, and shade $\frac{5}{100}$ meter.)

MP.6

T: (Point to the first $\frac{1}{10}$ meter shaded.) How many hundredths of a meter are shaded here?

S: $\frac{10}{100}$ meter.

T: (Point to the second $\frac{1}{10}$ meter shaded.) How many hundredths of a meter are shaded here?

S: $\frac{10}{100}$ meter.

T: How many hundredths of a meter are shaded altogether? Explain your thinking.

S: $\frac{25}{100}$ meter. → I see $\frac{10}{100}$ meter in each of the first two parts that were shaded. That is $\frac{20}{100}$ meter. Then, we shaded $\frac{5}{100}$ meter more. $\frac{20}{100}$ m + $\frac{5}{100}$ m = $\frac{25}{100}$ m.

T: (Write 0.25.) 25 hundredths can be written as a decimal in this way.

> **NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:**
>
> Some learners may find partitioning hundredths on the meter strip challenging. Alternatively, have students model with an area model (e.g., a 10 by 10 square partitioned into 100 unit squares). Or, enlarge the template (and tape diagrams on the Problem Set) to ease the task for students working below grade level and others. It may be helpful to use color to help students read hundredths on the Problem Set.

1 meter

Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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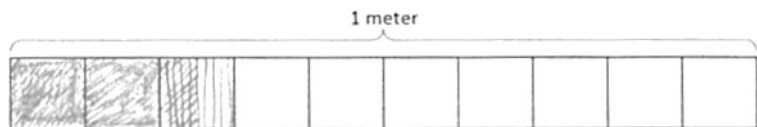
MP.6

T: (Make a number bond as shown to the right.) So, 25 hundredths is made of 2 tenths and…?

S: 5 hundredths.

T: (Write $\frac{2}{10}+\frac{5}{100}=\frac{25}{100}$.) Explain to your partner why this is true.

S: 2 tenths is the same as 20 hundredths, so it is the same as $\frac{20}{100}+\frac{5}{100}$. → 2 tenths is the same as $\frac{1}{10}+\frac{1}{10}$, and each tenth is $\frac{10}{100}$. So, $\frac{10}{100}+\frac{10}{100}+\frac{5}{100}=\frac{25}{100}$.
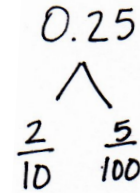
Have students continue by writing the total as a decimal and in a number bond to represent the tenths and hundredths fractions that compose the following decimals:

- 28 hundredths
- 31 hundredths
- 41 hundredths
- 79 hundredths

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 2(b), you showed that $\frac{1}{10}$ m = $\frac{10}{100}$ m. Write each number in decimal form. What do you notice?

Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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- Look at Problem 4(a). You shaded $\frac{8}{10}$ meter on a tape diagram. Can this be named in any other way? Use a diagram to explain your thinking, and show that number in decimal form.

- Share your number bond for Problem 3(b). How could you write this number bond showing both parts as hundredths? Why is it easier to show as much of the tape diagram as tenths as you can?

- Look at Problem 3(c). Why did we partition the fourth tenth into hundredths but left the first three tenths without partitioning?

- In Problem 5, how did you know how many tenths you could take out of the hundredths to make each number bond? Use a specific example to explain your reasoning.

- How do *hundredths* enable us to measure and communicate more precisely than tenths?

- Explain how hundredths are different from tenths.

- Refer to your solution for the Application Problem, and rename your answer using hundredths.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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A

Number Correct: _____

Write Fractions and Decimals

#			#		
1.	$\frac{2}{10}$ =	.	23.	1 =	$\frac{}{10}$
2.	$\frac{3}{10}$ =	.	24.	2 =	$\frac{}{10}$
3.	$\frac{4}{10}$ =	.	25.	5 =	$\frac{}{10}$
4.	$\frac{8}{10}$ =	.	26.	4 =	$\frac{}{10}$
5.	$\frac{6}{10}$ =	.	27.	4.1 =	$\frac{}{10}$
6.	0.1 =	$\frac{}{10}$	28.	4.2 =	$\frac{}{10}$
7.	0.2 =	$\frac{}{10}$	29.	4.6 =	$\frac{}{10}$
8.	0.3 =	$\frac{}{10}$	30.	2.6 =	$\frac{}{10}$
9.	0.7 =	$\frac{}{10}$	31.	3.6 =	$\frac{}{10}$
10.	0.5 =	$\frac{}{10}$	32.	3.4 =	$\frac{}{10}$
11.	$\frac{5}{10}$ =	.	33.	2.3 =	$\frac{}{10}$
12.	0.8 =	$\frac{}{10}$	34.	$4\frac{3}{10}$ =	.
13.	$\frac{7}{10}$ =	.	35.	$\frac{20}{10}$ =	.
14.	0.4 =	$\frac{}{10}$	36.	1.8 =	$\frac{}{10}$
15.	$\frac{9}{10}$ =	.	37.	$3\frac{4}{10}$ =	.
16.	$\frac{10}{10}$ =	.	38.	$\frac{50}{10}$ =	.
17.	$\frac{11}{10}$ =	.	39.	4.7 =	$\frac{}{10}$
18.	$\frac{12}{10}$ =	.	40.	$2\frac{8}{10}$ =	.
19.	$\frac{15}{10}$ =	.	41.	$\frac{30}{10}$ =	.
20.	$\frac{25}{10}$ =	.	42.	3.2 =	$\frac{}{10}$
21.	$\frac{45}{10}$ =	.	43.	$\frac{20}{10}$ =	.
22.	$\frac{38}{10}$ =	.	44.	2.1 =	$\frac{}{10}$

Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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B

Number Correct: _____

Improvement: _____

Write Fractions and Decimals

1.	$\frac{1}{10} =$	.	23.	$1 =$	$\frac{\ }{10}$
2.	$\frac{2}{10} =$	.	24.	$2 =$	$\frac{\ }{10}$
3.	$\frac{3}{10} =$	.	25.	$4 =$	$\frac{\ }{10}$
4.	$\frac{7}{10} =$	.	26.	$3 =$	$\frac{\ }{10}$
5.	$\frac{5}{10} =$	.	27.	$3.1 =$	$\frac{\ }{10}$
6.	$0.2 =$	$\frac{\ }{10}$	28.	$3.2 =$	$\frac{\ }{10}$
7.	$0.3 =$	$\frac{\ }{10}$	29.	$3.6 =$	$\frac{\ }{10}$
8.	$0.4 =$	$\frac{\ }{10}$	30.	$1.6 =$	$\frac{\ }{10}$
9.	$0.8 =$	$\frac{\ }{10}$	31.	$2.6 =$	$\frac{\ }{10}$
10.	$0.6 =$	$\frac{\ }{10}$	32.	$4.2 =$	$\frac{\ }{10}$
11.	$\frac{4}{10} =$	.	33.	$2.5 =$	$\frac{\ }{10}$
12.	$0.9 =$	$\frac{\ }{10}$	34.	$3\frac{4}{10} =$	.
13.	$\frac{6}{10} =$	.	35.	$\frac{50}{10} =$	.
14.	$0.5 =$	$\frac{\ }{10}$	36.	$1.7 =$	$\frac{\ }{10}$
15.	$\frac{9}{10} =$	.	37.	$4\frac{3}{10} =$	.
16.	$\frac{10}{10} =$	.	38.	$\frac{20}{10} =$	.
17.	$\frac{11}{10} =$	.	39.	$4.6 =$	$\frac{\ }{10}$
18.	$\frac{12}{10} =$	.	40.	$2\frac{4}{10} =$	.
19.	$\frac{17}{10} =$	.	41.	$\frac{40}{10} =$	.
20.	$\frac{27}{10} =$	.	42.	$2.3 =$	$\frac{\ }{10}$
21.	$\frac{47}{10} =$	.	43.	$\frac{30}{10} =$	.
22.	$\frac{34}{10} =$	.	44.	$4.1 =$	$\frac{\ }{10}$

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Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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Name _____   Date _____

1. a. What is the length of the shaded part
        of the meter stick in centimeters?

1 meter

    b. What fraction of a meter is 1 centimeter?

1 meter

    c. In fraction form, express the length of
        the shaded portion of the meter stick.

    d. In decimal form, express the length of the shaded portion of the meter stick.
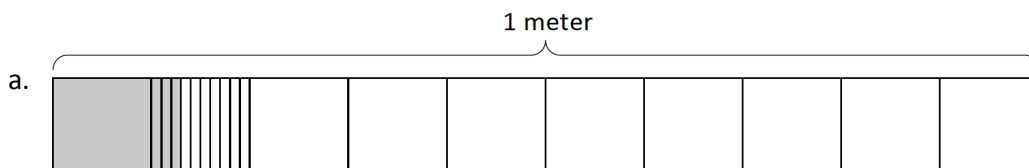
    e. What fraction of a meter is 10 centimeters?

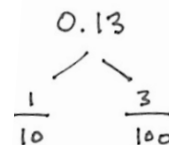2. Fill in the blanks.

    a. 1 tenth = _____ hundredths

    b. $\frac{1}{10}$ m = $\frac{}{100}$ m
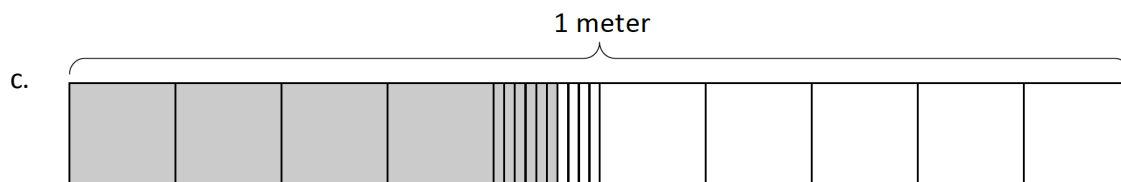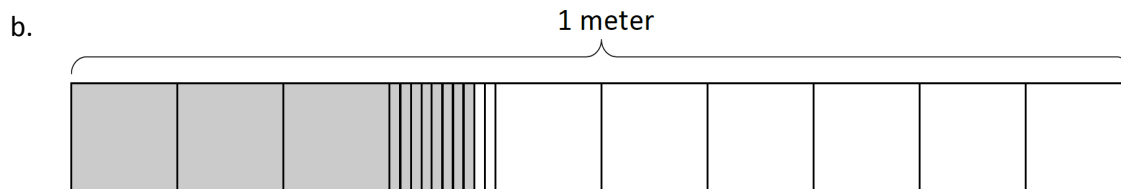
    c. $\frac{2}{10}$ m = $\frac{20}{}$ m

3. Use the model to add the shaded parts as shown. Write a number bond with the total written in decimal
   form and the parts written as fractions. The first one has been done for you.

    a.

1 meter

0.13

$\frac{1}{10}$ $\frac{3}{100}$

$$\frac{1}{10} \text{ m} + \frac{3}{100}\text{m} = \frac{13}{100}\text{m} = 0.13 \text{ m}$$

Lesson 4:   Use meters to model the decomposition of one whole into hundredths.
Represent and count hundredths.

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b.

1 meter

c.

1 meter

4. On each meter stick, shade in the amount shown. Then, write the equivalent decimal.
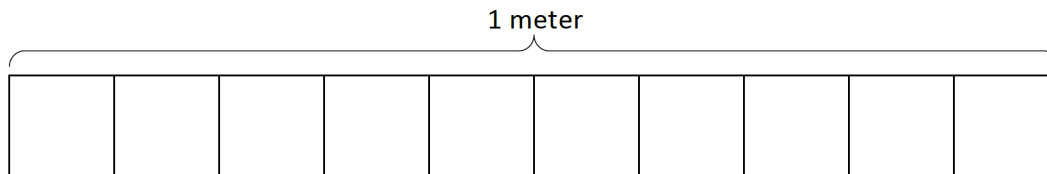
1 meter

a. $\frac{8}{10}$ m
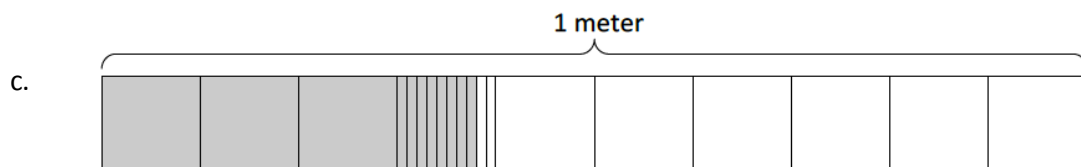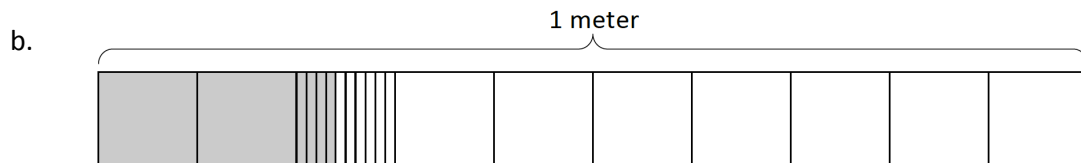
1 meter

b. $\frac{7}{100}$ m

1 meter
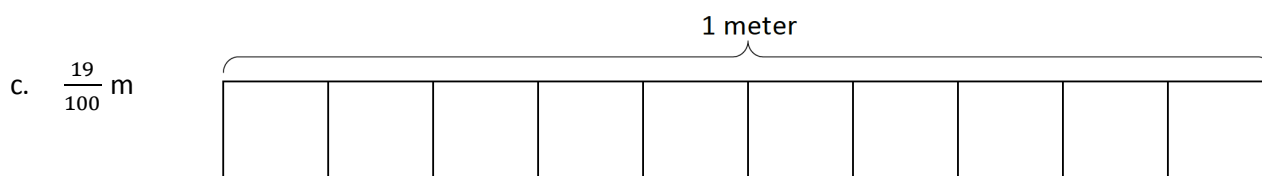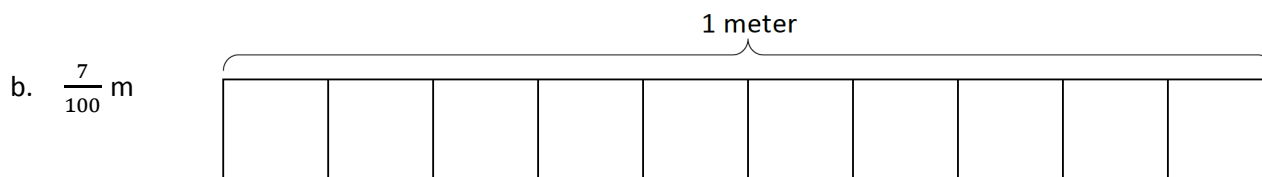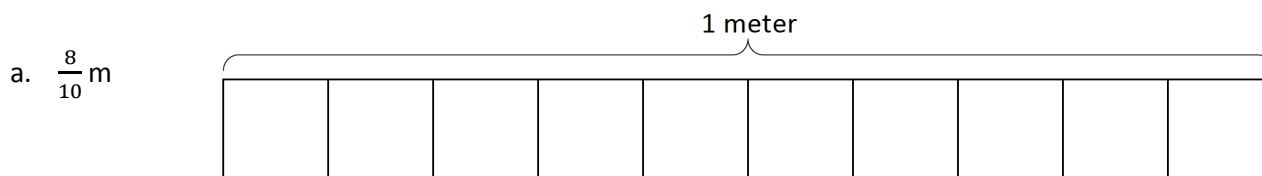
c. $\frac{19}{100}$ m

5. Draw a number bond, pulling out the tenths from the hundredths as in Problem 3. Write the total as the equivalent decimal.
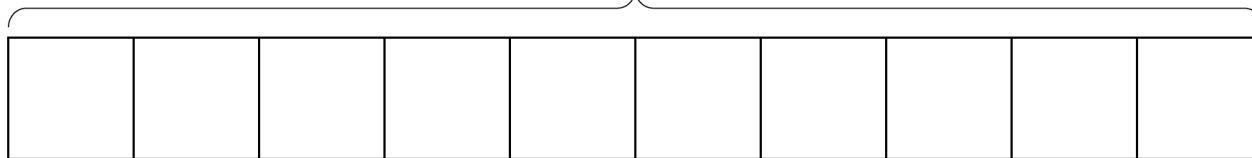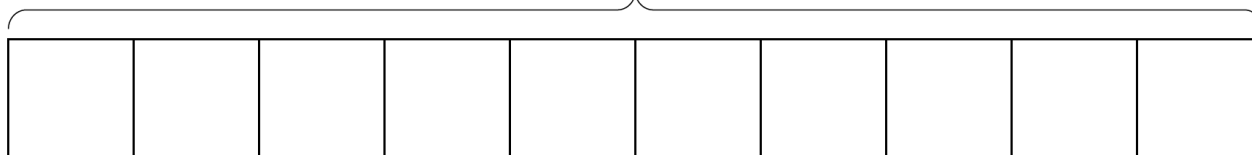
a. $\frac{19}{100}$ m

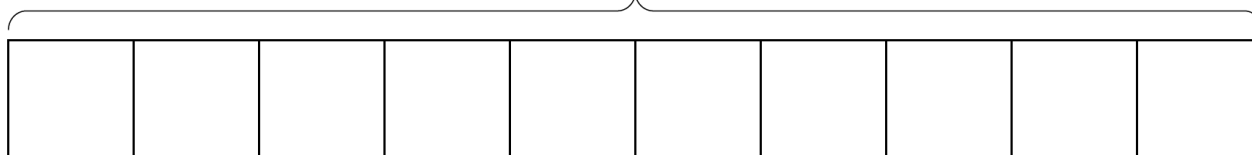b. $\frac{28}{100}$ m

c. $\frac{77}{100}$

d. $\frac{94}{100}$

Name _____   Date _____

1. Shade in the amount shown. Then, write the equivalent decimal.

1 meter

$\frac{6}{10}$ m
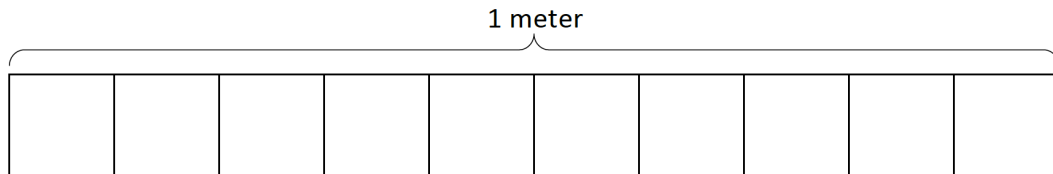
2. Draw a number bond, pulling out the tenths from the hundredths. Write the total as the equivalent decimal.

    a. $\frac{62}{100}$ m

    b. $\frac{27}{100}$

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Lesson 4: Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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Name _____   Date _____

1. a. What is the length of the shaded part of the meter stick in centimeters?
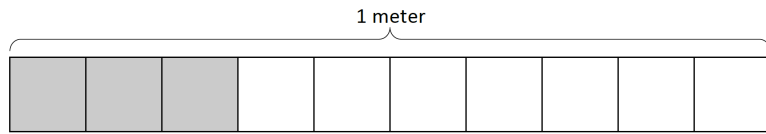
1 meter

b. What fraction of a meter is 3 centimeters?

c. In fraction form, express the length of the shaded portion of the meter stick.
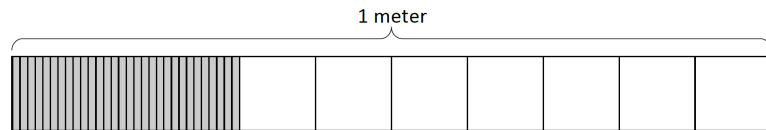
1 meter

d. In decimal form, express the length of the shaded portion of the meter stick.

e. What fraction of a meter is 30 centimeters?

2. Fill in the blanks.

a. 5 tenths = _____ hundredths

b. $\frac{5}{10}$ m = $\frac{}{100}$ m

c. $\frac{4}{10}$ m = $\frac{40}{}$ m

3. Use the model to add the shaded parts as shown. Write a number bond with the total written in decimal form and the parts written as fractions. The first one has been done for you.

1 meter

a.

0.13

$\frac{1}{10}$ $\frac{3}{100}$

$\frac{1}{10}$ m + $\frac{3}{100}$ m = $\frac{13}{100}$ m = 0.13 m

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Lesson 4:   Use meters to model the decomposition of one whole into hundredths. Represent and count hundredths.

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b.

1 meter

c.

1 meter

4. On each meter stick, shade in the amount shown. Then, write the equivalent decimal.

a. $\frac{9}{10}$ m
1 meter
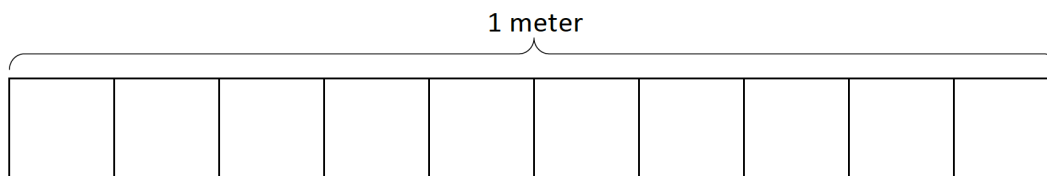
b. $\frac{15}{100}$ m
1 meter
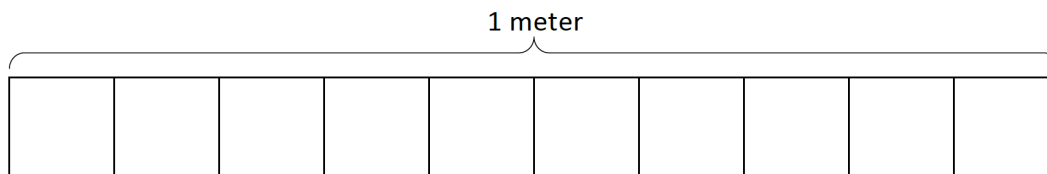
c. $\frac{41}{100}$ m
1 meter

5. Draw a number bond, pulling out the tenths from the hundredths, as in Problem 3 of the Homework.
    Write the total as the equivalent decimal.

a. $\frac{23}{100}$ m

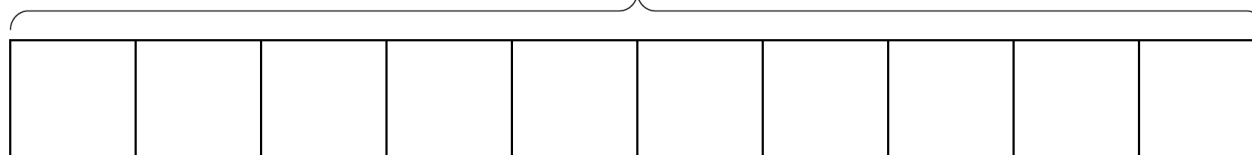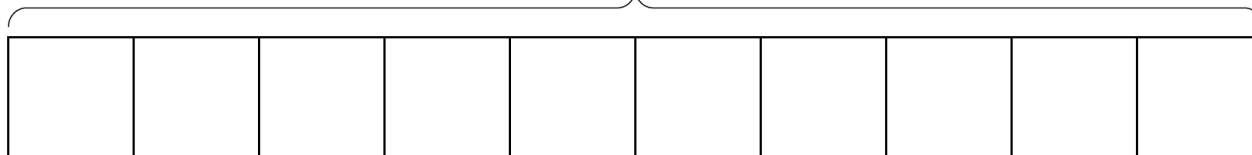b. $\frac{38}{100}$ m

c. $\frac{82}{100}$

d. $\frac{76}{100}$

Lesson 4:   Use meters to model the decomposition of one whole into hundredths.
Represent and count hundredths.

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1 meter

1 meter

1 meter

1 meter

1 meter

tape diagram in tenths

Lesson 5

Objective: Model the equivalence of tenths and hundredths using the area model and place value disks.

Suggested Lesson Structure

- ■ Fluency Practice   (12 minutes)
- ■ Application Problem   (6 minutes)
- ■ Concept Development   (32 minutes)
- ■ Student Debrief   (10 minutes)
  - **Total Time**   **(60 minutes)**
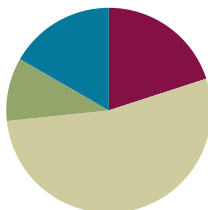
Fluency Practice (12 minutes)

- ▪ Divide by 10 **4.NF.7**   (3 minutes)
- ▪ Write the Decimal or Fraction **4.NF.5**   (4 minutes)
- ▪ Count by Tenths and Hundredths **4.NF.6**   (5 minutes)

Divide by 10 (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 4.

- T: (Project one 1 hundred disk. Beneath it, write 100 = 10 ___.) 100 is the same as 10 of what unit? Write the number sentence.
- S: (Write 100 = 10 tens.)
- T: (Write 100 = 10 tens.)

Continue with the following possible sequence: 10 = 10 ones, 1 = 10 tenths, and $\frac{1}{10}$ = 10 hundredths.

Write the Decimal or Fraction (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 4.

- T: (Write $\frac{1}{100}$.) Say the fraction.
- S: 1 hundredth.

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Lesson 5: Model the equivalence of tenths and hundredths using the area mode and place value disks.

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T: (Write $\frac{1}{100}$ = __.__.) Complete the number sentence.

S: (Write $\frac{1}{100}$ = 0.01.)

Continue with the following possible sequence: $\frac{2}{100}$, $\frac{3}{100}$, $\frac{7}{100}$, and $\frac{17}{100}$.

T: (Write $\frac{17}{100}$ = $\frac{10}{100}$ + $\frac{}{100}$ = 0.17.) Complete the number sentence.

S: (Write $\frac{17}{100}$ = $\frac{10}{100}$ + $\frac{7}{100}$ = 0.17.)

Continue with the following possible sequence: $\frac{13}{100}$ and $\frac{19}{100}$.

T: (Write 0.05 = ―.) Complete the number sentence.

S: (Write 0.05 = $\frac{5}{100}$.)

Continue with the following possible sequence: 0.15, 0.03, and 0.13.

T: (Write $\frac{100}{100}$.) Say the fraction.

S: 100 hundredths.

T: Complete the number sentence, writing 100 hundredths as a whole number.

S: (Write $\frac{100}{100}$ = 1.)

Count by Tenths and Hundredths (5 minutes)

Note: This fluency activity reviews Lessons 1 and 4.

T: 1 is the same as how many tenths?

S: 10 tenths.

T: Let's count to 10 tenths. When you come to 1, say 1.

S: $\frac{0}{10}$, $\frac{1}{10}$, $\frac{2}{10}$, $\frac{3}{10}$, $\frac{4}{10}$, $\frac{5}{10}$, $\frac{6}{10}$, $\frac{7}{10}$, $\frac{8}{10}$, $\frac{9}{10}$, 1.

T: Count by hundredths to 10 hundredths, starting at 0 hundredths.

S: $\frac{0}{100}$, $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, $\frac{4}{100}$, $\frac{5}{100}$, $\frac{6}{100}$, $\frac{7}{100}$, $\frac{8}{100}$, $\frac{9}{100}$, $\frac{10}{100}$.

T: 10 hundredths is the same as 1 of what unit?

S: 1 tenth.

T: Let's count to 10 hundredths again. This time, when you come to 1 tenth, say 1 tenth.

S: $\frac{0}{100}$, $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, $\frac{4}{100}$, $\frac{5}{100}$, $\frac{6}{100}$, $\frac{7}{100}$, $\frac{8}{100}$, $\frac{9}{100}$, $\frac{1}{10}$.

T: Count by hundredths again. This time, when I raise my hand, stop.

S: $\frac{0}{100}$, $\frac{1}{100}$, $\frac{2}{100}$, $\frac{3}{100}$, $\frac{4}{100}$.

NOTES ON MULTIPLE MEANS OF REPRESENTATION:

Distinguish *tenths* from *tens* for English language learners and others. Some students may not be able to differentiate the *th* sound at the end of the fraction words from the *s* sound at the end of *tens*. If possible, couple Count by Tenths and Hundredths with a visual aid, such as the fraction form, decimal form, or area model.

Lesson 5:   Model the equivalence of tenths and hundredths using the area mode and place value disks.

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T: (Raise hand.) Say 4 hundredths using digits.

S: Zero point zero 4.

T: Continue.

S: $\frac{5}{100}, \frac{6}{100}, \frac{7}{100}, \frac{8}{100}$.

T: (Raise hand.) Say 8 hundredths using digits.

S: Zero point zero 8.

T: Continue.

S: $\frac{9}{100}, \frac{1}{10}$.

T: Count backward by hundredths starting at 1 tenth.

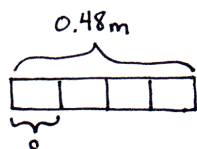Continue interrupting to express the hundredths using digits.

> **NOTES ON READING DECIMALS:**
>
> Students benefit from hearing decimal numbers read in both fraction form and as, for example, "zero point zero eight." Without the latter, it is hard to verify orally that students have written a decimal correctly. Furthermore, this manner of communicating decimals is used at times in the culture.
>
> However, saying "zero point zero eight" is the exception rather than the rule because "8 hundredths" communicates the equality of the fraction and decimal forms. The general rule is that students should read 0.08 and $\frac{8}{100}$ as 8 hundredths.

Application Problem (6 minutes)

The perimeter of a square measures 0.48 m. What is the measure of each side length in centimeters?

Note: The Application Problem reviews solving for an unknown side length (Module 4) and metric conversions (Module 2). Division of decimals is a Grade 5 standard, so instead, students might convert to centimeters (as in Solution A), use their fraction knowledge to decompose 48 hundredths into 4 equal parts (as in Solution B), or simply think in unit form (i.e., 48 hundredths ÷ 4 = 12 hundredths).

Concept Development (32 minutes)

Materials: (T) Tenths and hundredths area model (Template), tape diagram in tenths (Lesson 4 Template), decimal place value disks (S) Tenths and hundredths area model (Template), personal white board

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Lesson 5: Model the equivalence of tenths and hundredths using the area mode and place value disks.

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Problem 1: Simplify hundredths by division.

T: We can show the equivalence of 10 hundredths and 1 tenth in the same way we showed the equivalence of 2 fourths and 1 half by using division.

T: Shade 1 tenth of the first area model. Next, shade 10 hundredths on the second area model. Label each area model. What do you notice?

S: The same amount is shaded for each. → One area is decomposed into tenths and the other into hundredths, but the same amount is selected. That means they are equivalent.

T: (Write $\frac{1}{10} = \frac{10}{100}$.) Write the equivalent statement using decimals.

S: (Write 0.1 = 0.10.)

T: Show in the next area models how many tenths are equal to 30 hundredths. Write two equivalent statements using fractions and decimals.

S: (Shade area models.) $\frac{3}{10} = \frac{30}{100}$. 0.3 = 0.30.

T: Let's show those as equivalent fractions using division. (Write $\frac{10}{100} = \frac{10 \div 10}{100 \div 10} = \frac{1}{10}$.) Why did I divide by 10?

S: It is a common factor of 10 and 100. → Dividing the denominator by 10 gives us tenths, and we are showing equivalent fractions for tenths and hundredths. → We can make a larger unit from 10 hundredths.

T: With your partner, use division to find how many tenths are equal to 30 hundredths.

S: (Record $\frac{30}{100} = \frac{30 \div 10}{100 \div 10} = \frac{3}{10}$.) 3 tenths.

T: With your partner, use multiplication to find how many hundredths are in 3 tenths.

S: (Record $\frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100}$.) 30 hundredths.

T: Is there a pattern as you find equivalent fractions for tenths and hundredths?

S: I multiply the number of tenths by 10 to get the number of hundredths, and I divide the number of hundredths by 10 to get the number of tenths. → I can convert tenths to hundredths in my head by putting a zero at the end of the numerator and denominator. I can convert hundredths to tenths by removing a zero from the numerator and denominator. → We are just changing the units, making either larger or smaller units. Both have the same value.

MP.8

Have students convert 7 tenths to 70 hundredths using multiplication and 70 hundredths to 7 tenths using division.

$$\frac{10}{100} = \frac{10 \div 10}{100 \div 10} = \frac{1}{10}$$

$$\frac{30}{100} = \frac{30 \div 10}{100 \div 10} = \frac{3}{10}$$

$$\frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100}$$

$\frac{30}{100} = \frac{3}{10}$ just like $\frac{2}{4} = \frac{1}{2}$

$\frac{30 \div 10}{100 \div 10} = \frac{3}{10}$ just like $\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

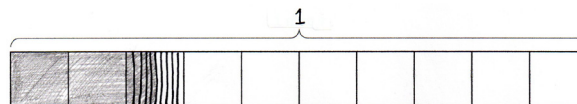We just make larger units!

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Problem 2: Model hundredths with an area model.

T: (Project a tape diagram, as was used in Lesson 4, with $\frac{25}{100}$ shaded.) Say the fractional part that is shaded.

S: 25 hundredths.

T: Say it as a decimal number.

S: 25 hundredths. We say it the same way.

T: Yes. Both the fraction and decimal number represent the same amount. What is different is the way that they are written. Write 25 hundredths as a fraction and then as a decimal number.

S: (Write $\frac{25}{100}$ and 0.25.)

T: Just as we can express 25 hundredths in different ways when we write it, we can also represent it in different ways pictorially, just like we did with tenths and other fractions from Module 5. (Project the area model.) How can we shade $\frac{25}{100}$?

S: We can draw horizontal lines to make smaller units. → We can decompose each tenth into 10 parts to make hundredths using horizontal lines.

T: Yes. Decimals like this are just fractions. We are doing exactly the same thing, but we are writing the number in a different way. Go ahead and make the hundredths.

S: (Partition the area model.)

T: Shade $\frac{25}{100}$. (Allow students time to shade the area.)

T: What is a shortcut for shading 25 hundredths?

S: There are 10 hundredths in each column. I shaded 10 hundredths at a time. $\frac{10}{100}$, $\frac{20}{100}$. Then, I shaded 5 hundredths more. → I shaded 2 columns and then 5 more units. → A tenth, and a tenth and 5 hundredths. → I shaded two and a half columns.
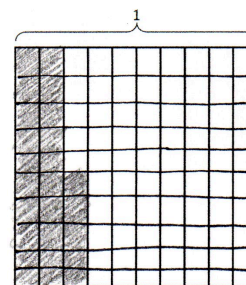
T: In total, how many tenths are shaded?

S: 2 tenths and part of another tenth. → $2\frac{1}{2}$ tenths.

T: Both are correct: 2 complete tenths are shaded, and another half of a tenth is shaded. In total, how many hundredths are shaded?

S: 25 hundredths.

Repeat with $\frac{52}{100}$ and $\frac{35}{100}$.

Problem 3: Compose hundredths to tenths using place value disks, and then represent with a number bond.

T: Look at the area model we just drew. 1 tenth equals how many hundredths?

S: 10 hundredths.

T: Write it in decimal form.

S: 0.10. → 0.1.

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Lesson 5: Model the equivalence of tenths and hundredths using the area mode and place value disks.

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T: (Project 16 hundredths with place value disks.) What is the value of each disk? How can you tell?

S: 1 hundredth. I see point zero one on each disk.

T: How many hundredths are there?
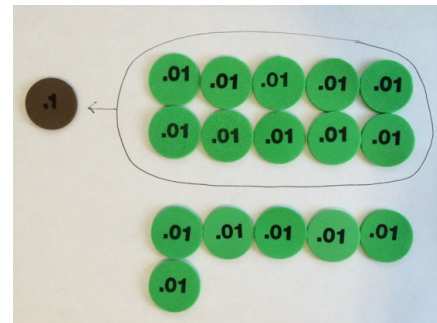
S: 16 hundredths.

T: Can we make a tenth? Talk to your partner.

S: 10 hundredths can be traded for 1 tenth. → Yes! We can compose 10 hundredths to 1 tenth since $\frac{1}{10} = \frac{10}{100}$. → It is just like place value: 10 ones make 1 ten, or 10 tens make 1 hundred.

T: Circle 10 hundredths to show 1 tenth. What is represented now?

S: 1 tenth and 6 hundredths.

T: (Draw a number bond to show the parts of 1 tenth and 6 hundredths. Point to the number bond.) 16 hundredths can be represented as 1 tenth and 6 hundredths.

Repeat with 13 hundredths and 22 hundredths.

Problem 4: Use place value disks to represent a decimal fraction. Write the equivalent decimal in unit form.

T: (Write $\frac{5}{100}$.) Draw place value disks to represent this fraction.

S: (Draw 5 hundredths disks.)
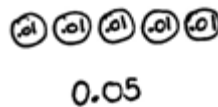
T: Say it in unit form.

S: 5 hundredths.

T: Write it as a decimal. Be careful that your decimal notation shows hundredths.

S: (Write 0.05.)

T: (Write $\frac{25}{100}$.) Draw place value disks to represent this fraction.

S: That is 25 hundredths! → We can represent $\frac{25}{100}$ with 2 tenth disks and 5 hundredth disks.

T: I hope so, since it takes much too long to draw 25 hundredths. Say the number in unit form, and write it as a decimal.

S: 25 hundredths. → 0.25.

Repeat with 32 hundredths and 64 hundredths.

NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:

Students working below grade level and English language learners may benefit from additional practice reading and writing decimals. If students are confusing the decimal notation (for example, modeling 0.5 rather than 0.05), couple place value disks with the area model, and have students count and recount their disks.
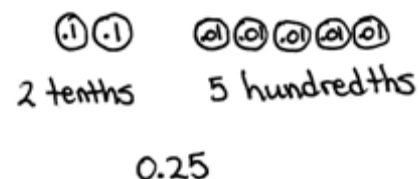
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Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Model the equivalence of tenths and hundredths using the area model and place value disks.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How does solving Problem 1(a) help you solve Problem 2(a)?

- In Problem 3(a), how does circling groups of 10 hundredths help you find how many tenths are in the number?

- In Problem 4(a), how did you write 3 hundredths in decimal form? A student wrote 0.3 (zero point 3). What number did she write? Use your disks to explain how to properly express 3 hundredths in decimal form.

- With your partner, compare the answers to Problem 4 (d) and (f). Did you write the same equivalent numbers? Why are there several possibilities for answers in these two problems? Where have we seen that before?

- How is using the area model to show tenths and hundredths similar to or different from using place value disks to show tenths and hundredths? Which model do you prefer and why?

Lesson 5:   Model the equivalence of tenths and hundredths using the area mode and place value disks.

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- ▪ How is exchanging 10 hundredths for 1 tenth like exchanging 10 tens for 1 hundred? How is it different?
- ▪ Use an area model to model both renaming 3 sixths as 1 half and renaming 30 hundredths as 3 tenths. What is happening to the units in both renamings?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Lesson 5:   Model the equivalence of tenths and hundredths using the area mode and place value disks.

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Name _____   Date _____

1. Find the equivalent fraction using multiplication or division. Shade the area models to show the equivalency. Record it as a decimal.

   a. $\dfrac{3 \times}{10 \times} = \dfrac{}{100}$

   b. $\dfrac{50 \div}{100 \div} = \dfrac{}{10}$
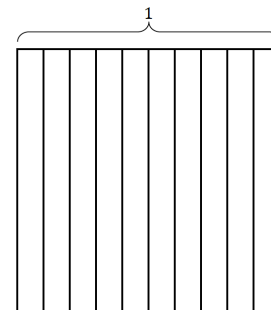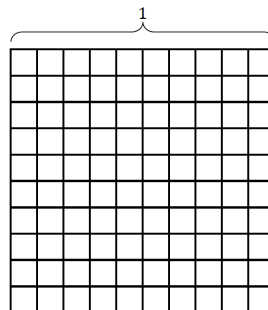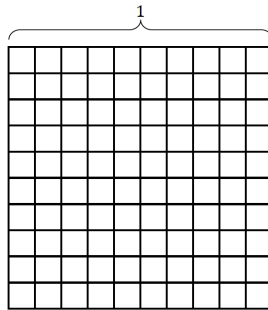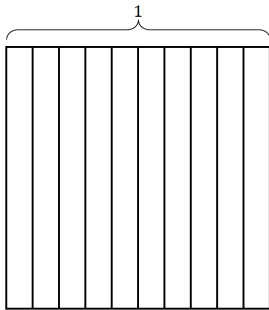
2. Complete the number sentences. Shade the equivalent amount on the area model, drawing horizontal lines to make hundredths.

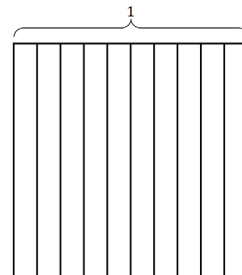   a. 37 hundredths = _____ tenths + _____ hundredths
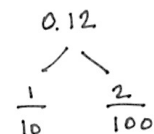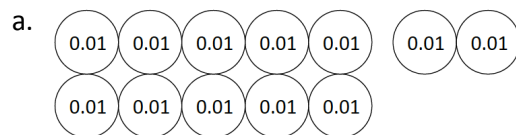
      Fraction form: _____
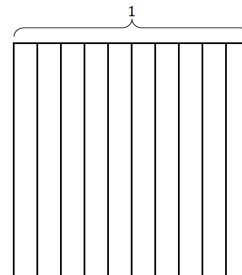
      Decimal form: _____

   b. 75 hundredths = _____ tenths + _____ hundredths

      Fraction form: _____

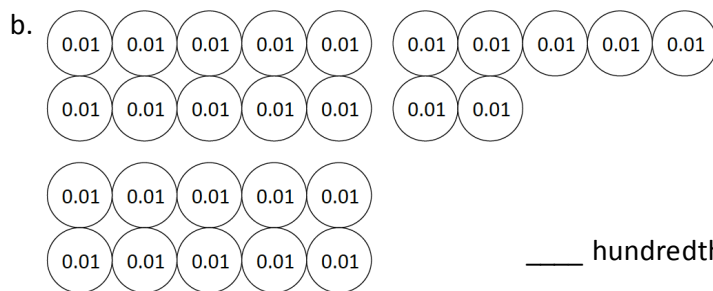   Decimal form: _____

3. Circle hundredths to compose as many tenths as you can. Complete the number sentences. Represent each with a number bond as shown.

   a.

   _____ hundredths = _____ tenth + _____ hundredths

Lesson 5: | Model the equivalence of tenths and hundredths using the area mode and place value disks.

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b.

(0.01)(0.01)(0.01)(0.01)(0.01)   (0.01)(0.01)(0.01)(0.01)(0.01)

(0.01)(0.01)(0.01)(0.01)(0.01)   (0.01)(0.01)

(0.01)(0.01)(0.01)(0.01)(0.01)

(0.01)(0.01)(0.01)(0.01)(0.01)   _____ hundredths = _____ tenths + _____ hundredths

4. Use both tenths and hundredths place value disks to represent each number. Write the equivalent number in decimal, fraction, and unit form.

a. $\frac{3}{100}$ = 0. _____ _____ hundredths	b. $\frac{15}{100}$ = 0. _____ _____ tenth _____ hundredths
c. ⎯ = 0.72 _____ hundredths	d. ⎯ = 0.80 _____ tenths
e. ⎯ = 0. _____ 7 tenths 2 hundredths	f. ⎯ = 0. _____ 80 hundredths

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Name _____   Date _____

Use both tenths and hundredths place value disks to represent each fraction. Write the equivalent decimal, and fill in the blanks to represent each in unit form.

1. $\frac{7}{100}$ = 0.____

       ____ hundredths

2. $\frac{34}{100}$ = 0. ____

       ____ tenths ____ hundredths

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Lesson 5:   Model the equivalence of tenths and hundredths using the area mode and place value disks.

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Name _____   Date _____

1. Find the equivalent fraction using multiplication or division. Shade the area models to show the equivalency. Record it as a decimal.

    a. $\dfrac{4 \times}{10 \times} = \dfrac{}{100}$
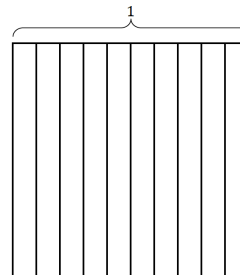
    b. $\dfrac{60 \div}{100 \div} = \dfrac{}{10}$
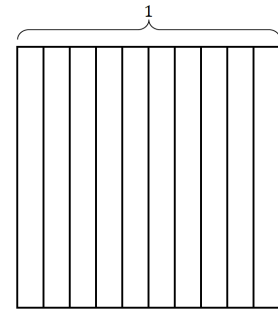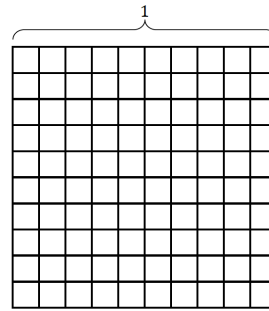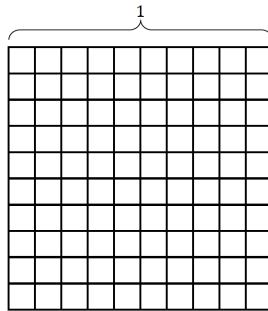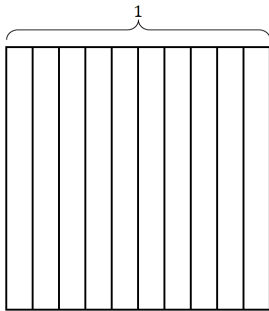
2. Complete the number sentences. Shade the equivalent amount on the area model, drawing horizontal lines to make hundredths.

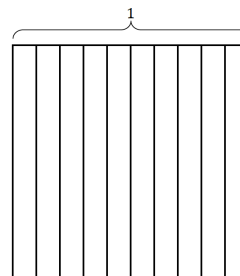   a. 36 hundredths = _____ tenths + _____ hundredths

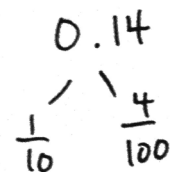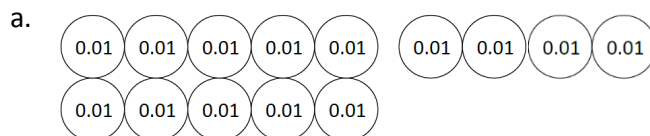   Decimal form: _____

        Fraction form: _____

    b. 82 hundredths = _____ tenths + _____ hundredths
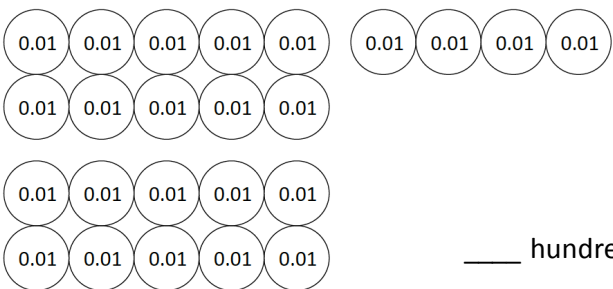
        Decimal form: _____

        Fraction form: _____

3. Circle hundredths to compose as many tenths as you can. Complete the number sentences. Represent each with a number bond as shown.

    a. (0.01) (0.01) (0.01) (0.01) (0.01)   (0.01) (0.01) (0.01) (0.01)

        (0.01) (0.01) (0.01) (0.01) (0.01)

        0.14

        $\dfrac{1}{10}$ $\dfrac{4}{100}$

        ____ hundredths = _____ tenth + _____ hundredths

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Lesson 5:   Model the equivalence of tenths and hundredths using the area model and place value disks.

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b.

| 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | | 0.01 | 0.01 | 0.01 | 0.01 |

| 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

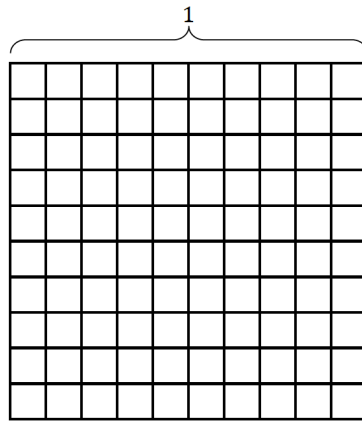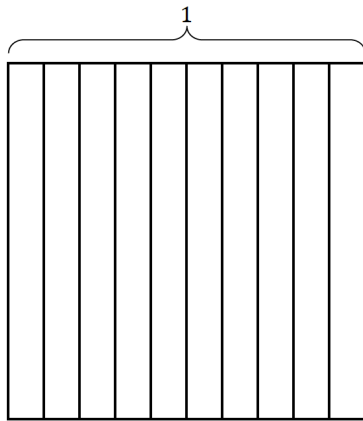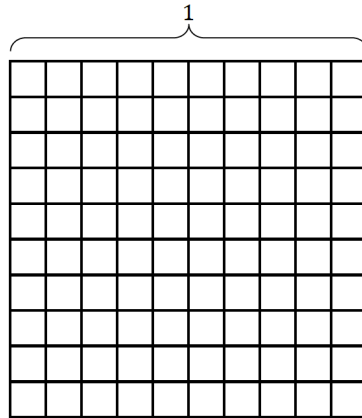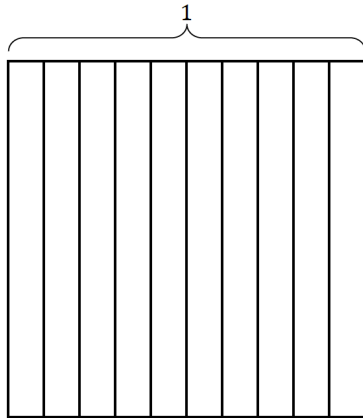| 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

| 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |

_____ hundredths = _____ tenths + _____ hundredths

4. Use both tenths and hundredths place value disks to represent each number. Write the equivalent number in decimal, fraction, and unit form.

a. $\frac{4}{100}$ = 0. _____ _____ hundredths	b. $\frac{13}{100}$ = 0. _____ _____ tenth _____ hundredths
c. —— = 0.41 _____ hundredths	d. —— = 0.90 _____ tenths
e. —— = 0. _____ 6 tenths 3 hundredths	f. —— = 0. _____ 90 hundredths

Lesson 5:   Model the equivalence of tenths and hundredths using the area mode and place value disks.

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tenths and hundredths area model

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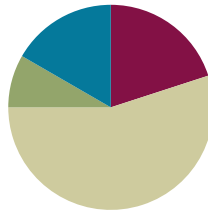
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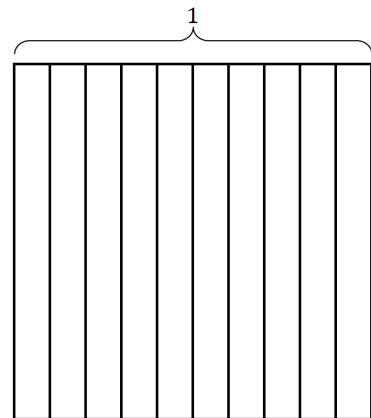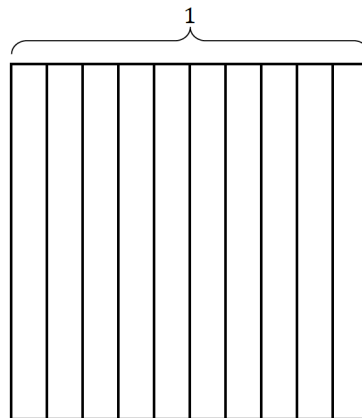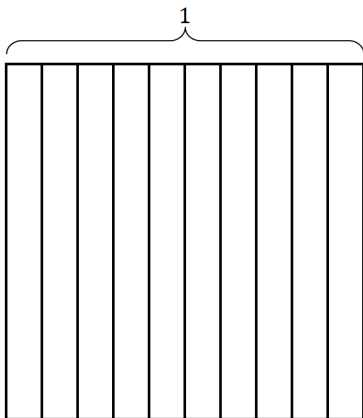
Lesson 6

Objective: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
■ Application Problem   (5 minutes)
■ Concept Development   (33 minutes)
■ Student Debrief   (10 minutes)
    Total Time   **(60 minutes)**

Fluency Practice (12 minutes)

▪ Count by Hundredths **4.NF.6**   (5 minutes)
▪ Write the Decimal or Fraction **4.NF.5**   (4 minutes)
▪ Break Apart Hundredths **4.NF.5**   (3 minutes)

Count by Hundredths (5 minutes)

Note: This fluency activity reviews Lessons 4–5.

T: Count by fives to 30, starting at zero.

S: 0, 5, 10, 15, 20, 25, 30.

T: Count by 5 hundredths to 30 hundredths, starting at 0 hundredths. (Write as students count.)

S: $\frac{0}{100}, \frac{5}{100}, \frac{10}{100}, \frac{15}{100}, \frac{20}{100}, \frac{25}{100}, \frac{30}{100}$.

$\frac{0}{100}$	$\frac{5}{100}$	$\frac{10}{100}$	$\frac{15}{100}$	$\frac{20}{100}$	$\frac{25}{100}$	$\frac{30}{100}$
$\frac{0}{10}$		$\frac{1}{10}$		$\frac{2}{10}$		$\frac{3}{10}$

T: 1 tenth is the same as how many hundredths?

S: 10 hundredths.

T: (Beneath $\frac{10}{100}$, write $\frac{1}{10}$.)

Continue the process for $\frac{2}{10}$ and $\frac{3}{10}$.

T: Let's count by 5 hundredths again. This time, when you come to a tenth, say the tenth. Try not to look at the board.

S: $\frac{0}{100}, \frac{5}{100}, \frac{1}{10}, \frac{15}{100}, \frac{2}{10}, \frac{25}{100}, \frac{3}{10}$.

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Lesson 6:   Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

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T: Count backward by 5 hundredths, starting at 3 tenths.

S: $\frac{3}{10}, \frac{25}{100}, \frac{2}{10}, \frac{15}{100}, \frac{1}{10}, \frac{5}{100}, \frac{0}{100}$.

T: Count by 5 hundredths again. This time, when I raise my hand, stop.

S: $\frac{0}{100}, \frac{5}{100}, \frac{1}{10}, \frac{15}{100}$.

T: (Raise hand.) Say 15 hundredths using digits.

S: Zero point one five.

T: Continue.

S: $\frac{2}{10}, \frac{25}{100}, \frac{3}{10}$.

T: (Raise hand.) Say 3 tenths in digits.

S: Zero point three.

T: Count backward starting at 3 tenths.

S: $\frac{3}{10}, \frac{25}{100}$.

T: (Raise hand.) Say 25 hundredths in digits.

S: Zero point two five.

T: Continue.

S: $\frac{2}{10}, \frac{15}{100}, \frac{1}{10}$.

T: (Raise hand.) Say 1 tenth in digits.

S: Zero point one.

T: Continue.

S: $\frac{5}{100}, \frac{0}{100}$.

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> Students working below grade level and others may find it challenging to integrate equivalent fractions (such as $\frac{1}{10}$) into the Count by Hundredths fluency activity. Ease the task by chunking. Count a little at a time, and repeat the count so that students are comfortable, confident, and excited. For example, lead students to count from $\frac{1}{100}$ to $\frac{1}{10}$, repeat a few times, and then add onto the count $\frac{15}{100}$, and so on.

Write the Decimal or Fraction (4 minutes)

Materials: (T) Hundredths area model (Fluency Template), personal white board (S) Personal white board

Note: This fluency activity reviews Lessons 4–5.

T: (Project the hundredths area model. Shade 3 units.) 1 whole is decomposed into 100 equal units. Write the fraction of the grid that is shaded.

S: (Write $\frac{3}{100}$.)

T: (Write $\frac{3}{100} = $ __.__.) Complete the number sentence.

S: (Write $\frac{3}{100} = 0.03$.)

Continue the process for $\frac{5}{100}, \frac{8}{100}, \frac{4}{100},$ and $\frac{14}{100}$.

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Lesson 6: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

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T: (Write $\frac{14}{100} = \frac{10}{100} + \frac{}{100} = 0.14$.) Complete the number sentence.

S: (Write $\frac{14}{100} = \frac{10}{100} + \frac{4}{100} = 0.14$.)

Continue with the following possible sequence: $\frac{17}{100}$ and $\frac{53}{100}$.

T: (Shade 4 units.) Write the amount of the grid that is shaded as a decimal.

S: (Write 0.04.)

T: (Write $0.04 = \frac{}{100}$.) Complete the number sentence.

S: (Write $0.04 = \frac{4}{100}$.)

Continue with the following possible sequence: 0.14, 0.06, and 0.16.

T: (Shade in the entire grid.) Write the amount of the grid that is shaded as a fraction and as a digit.

S: (Write $\frac{100}{100} = 1$.)

Break Apart Hundredths (3 minutes)

Materials: (T/S) Personal white board

Note: This fluency activity reviews Lesson 5.

T: (Project 13 hundredths disks.) Say the value.

S: 13 hundredths.

T: Write the value of the disks as a decimal.

S: (Write 0.13.)

T: (Write $0.13 = \text{—}$.) Write 13 hundredths as a fraction.

S: (Write $0.13 = \frac{13}{100}$.)

T: How many hundredths are in 1 tenth?

S: 10 hundredths.

T: Draw place value disks to represent the 13 hundredths after composing 1 tenth.

S: (Draw 1 tenth disk and 3 hundredth disks.)

T: (Write $0.13 = \frac{13}{100} = \frac{}{10} + \frac{}{100}$.) Complete the number sentence.

S: (Write $0.13 = \frac{13}{100} = \frac{1}{10} + \frac{3}{100}$.)

Continue with the following possible sequence: 0.21 and 0.14.

Lesson 6:   Use the area model and number line to represent mixed numbers with
units of ones, tenths, and hundredths in fraction and decimal forms.

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Application Problem (5 minutes)

The table shows the perimeter of four rectangles.

a. Which rectangle has the smallest perimeter?

b. The perimeter of Rectangle C is how many meters less than a kilometer?

c. Compare the perimeters of Rectangles B and D. Which rectangle has the greater perimeter? How much greater?

Rectangle	Perimeter
A	54 cm
B	$\frac{69}{100}$ m
C	54 m
D	0.8 m

a) Rectangle A has the smallest perimeter.

$A: 54\,cm = \boxed{\frac{54}{100}\,m}$

$B: \frac{69}{100}\,m$

$C: 54\,m$

$D: 0.8\,m = \frac{8}{10}\,m = \frac{80}{100}\,m$

b) The perimeter of Rectangle C is 946 meters less than a kilometer.

$1\,km = 1,000\,m$

$$\begin{array}{r} \overset{9\ 9\ 10}{\cancel{1000}} \\ -\quad 54 \\ \hline 946 \end{array}$$

c) Rectangle D's perimeter is $\frac{11}{100}$ m greater than Rectangle B's perimeter.

$\frac{80}{100} - \frac{69}{100} = \frac{11}{100}$

Note: This Application Problem reviews related metric units (Module 2) and comparing measurements expressed as fractions and decimals in preparation for work with mixed numbers, metric units, and place value in today's Concept Development.

Concept Development (33 minutes)

Materials: (T/S) Area model (Template 1), number line (Template 2), pencil, personal white board

Problem 1: Represent mixed numbers with units of ones, tenths, and hundredths using area models.

T: (Write $1\frac{22}{100}$.) How many ones?
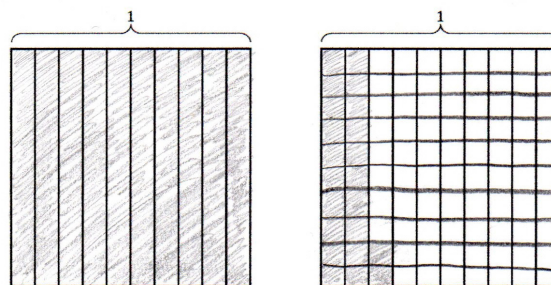
S: 1 one.

T: How many hundredths more than 1?

S: 22 hundredths.

T: (Distribute Template 1, area model.) Use the area models to shade $1\frac{22}{100}$.

S: (Shade the area models.)

T: How many ones are shaded?

S: 1 one.

Lesson 6: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

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T: What fraction of another one is shaded?

S: 22 hundredths.

T: Write $1\frac{22}{100}$ as a decimal number.

S: (Write 1.22.)

Continue with $1\frac{38}{100}, 1\frac{60}{100}$, and $1\frac{81}{100}$.

Problem 2: Represent mixed numbers with units of ones, tenths, and hundredths on a number line.

T: (Refer to the area models representing 1.22.) We have used tape diagrams, area models, and place value disks to represent decimal numbers. We can also use a number line. (Distribute Template 2, number line, and label the intervals of 0, 1, 2, and 3.) To find 1.22 on a number line, we can start with the largest unit. What is the largest unit?

S: Ones.

T: Start at zero, and slide 1 one. What is remaining?

S: 22 hundredths.

T: What is the next largest unit?

S: Tenths.

T: How many tenths?

S: 2 tenths.

T: From 1 one, slide 2 tenths. What remains?

S: 2 hundredths.

T: Can we show hundredths? How do we partition tenths into hundredths?

S: Each tenth would be split into 10 parts, just like on a tape diagram or an area model. It's hard to do that here because the tenths are so small.

T: Let's estimate where the hundredths would be. We need to show 2 hundredths. If I imagine each tenth partitioned into ten parts, where would 2 hundredths be? I will move very slowly. Say, "Stop!" when I get to 1 and 22 hundredths. (Slide very slowly from 1.2.)

S: Stop! (This should be at a place just beyond 1 and 2 tenths.)

T: Draw an arrow to show this very small slide. Discuss with a partner. How did we move from zero to 1.22?

S: We began with moving 1 one. Then, we moved 2 tenths, and then we moved 2 hundredths. → We started at zero and went up, beginning with the largest unit, the ones, the tenths, and then the hundredths. → We slid units from left to right, largest to smallest, but we estimated the 2 hundredths.

T: Draw a point to show where 1.22 is located. Write the number in decimal form.

T: Let's locate $3\frac{46}{100}$ on the next number line. Can we label the intervals the same?

S: No, because this point will come after 3.

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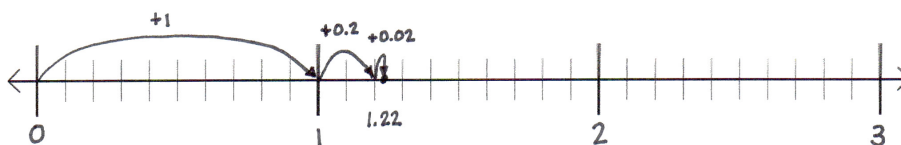
T: Start the number line at 3 ones. We will locate $\frac{46}{100}$ more than 3. Decompose $\frac{46}{100}$ into tenths and hundredths.

S: $\frac{46}{100} = \frac{4}{10} + \frac{6}{100}$.

T: Which unit is larger: tenths or hundredths?

S: Tenths.

T: Let's count up 4 tenths. Draw an arrow, or keep track of the movement with your pencil. Now, what unit is left?

S: Hundredths. We have 6 hundredths. 6 hundredths is one hundredth more than 5 hundredths, so 4 tenths 6 hundredths would be just past the midpoint of 4 tenths and 5 tenths.

T: Draw a point to show where $3\frac{46}{100}$ is located. Write the number in decimal form.

S: (Draw and write 3.46.)
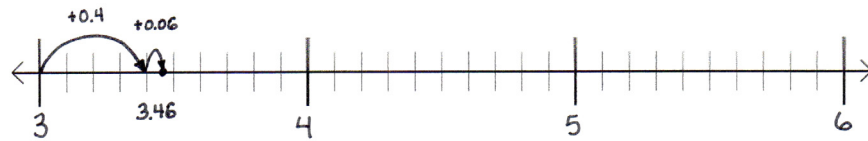
> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> Students working above grade level or others may present alternative ways of locating $3\frac{46}{100}$ on the number line, such as reasoning that half of 100 is 50 and then counting back to 46. Efficiency and variety in strategies are always welcome.

Repeat with 2.34 and 3.70.

Problem 3: Match the unit form of a mixed number to its decimal and fraction forms.

MP.6

T: When we write decimal numbers, the decimal point separates the whole number part on the left from the decimal fraction part on the right.

T: Write 3 ones 8 tenths as a decimal.

S: (Write 3.8.)

T: The ones and the tenths each have a special place. (Label each place value.)

T: Write 3 ones 8 hundredths in decimal form. Show your partner what you have written. Are your answers the same?

S: The answer is 3.8. → I disagree. That would be 3 ones 8 tenths. We want hundredths. It's 3.08. There are no tenths. We need to put a zero to show that. It's just like when we write whole numbers. The zero holds a place value.

T: Look again at 3 ones 8 tenths.

T: Place a zero to the right of the digit eight. Say that number in unit form.

S: 3 ones 80 hundredths.

T: Express 80 hundredths as tenths.

S: 8 tenths.

Lesson 6: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

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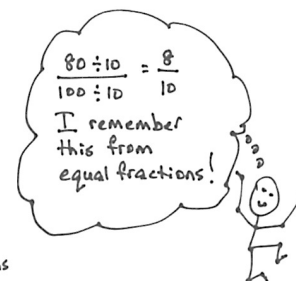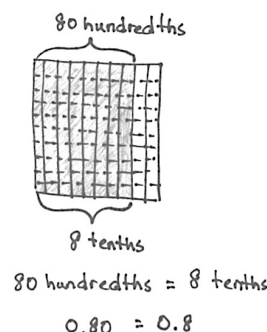
T: Yes. 0.80 and 0.8 are equivalent. We have shown this using an area model and using division, too, when the number was in fraction form.

T: Let's practice writing fractions and decimals. Be mindful of each digit's place in the number.

T: Write 2 ones 8 hundredths as a mixed number and then as a decimal number.

S: $2\frac{8}{100}$, 2.08.

T: Write 8 ones 2 hundredths as a mixed number and a decimal number.

S: $8\frac{2}{100}$, 8.2. Wait! That decimal is not right. That would be 8 and 2 tenths. It is 8.02. There are 8 ones, 0 tenths, and 2 hundredths.

Repeat, as needed, with 9 ones 80 hundredths, 2 ones 2 tenths, and 4 ones 7 hundredths.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How could you count backward to locate 2.47 on the number line in Problem 1(b)?

- In Problem 2(a), how did you estimate the location of your point?

- In Problem 3(a), the units are ones and hundredths. If I had 1.02 liters of water and you had 1.02 kilograms of rice, how do the measurement units change the meaning of that number?

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- In Problem 3(f), express this number in ones and tenths. Use a model to show that this new representation is equivalent to 7 ones 70 hundredths.

- Simplify $7\frac{70}{100}$ using division to show it is equal to $7\frac{7}{10}$. Explain to your partner how that relates to 7.70 = 7.7.

- Explain to your partner why there is one less item in the left and right columns of Problem 4 than in the center column.

- Compare. (Write 1.4 meters _____ 1.7 grams.) Does it make sense to compare meters with grams? Why not?

- Talk with your partner about the importance of the number zero. Use the number 100 and the number 0.01 in your discussion. (Provide Hide Zero cards to strengthen the conversation.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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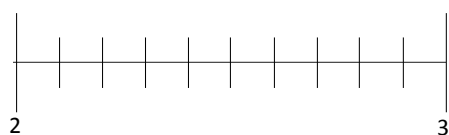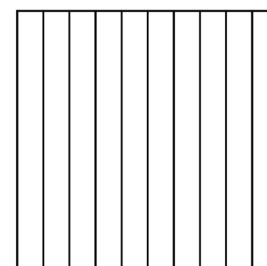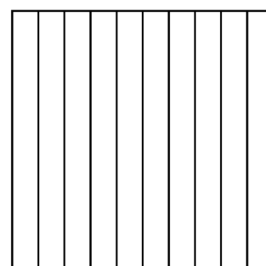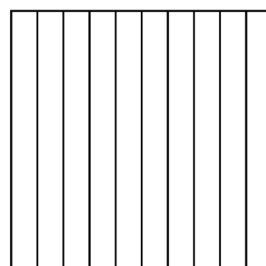Name _____ Date _____

1. Shade the area models to represent the number, drawing horizontal lines to make hundredths as needed. Locate the corresponding point on the number line. Label with a point, and record the mixed number as a decimal.

   a. $1\frac{15}{100} = $ ___._____
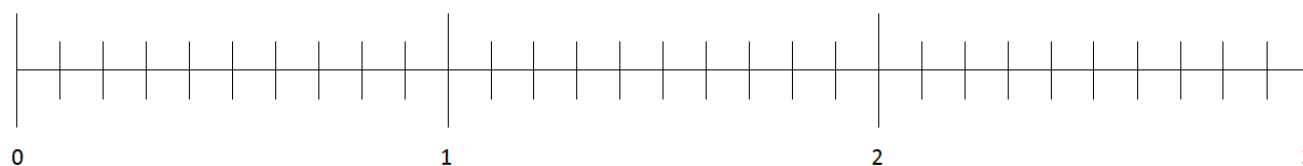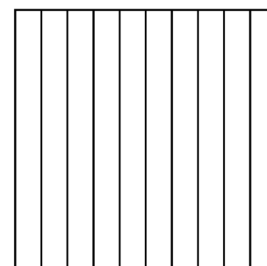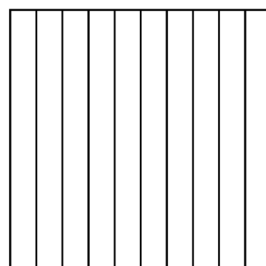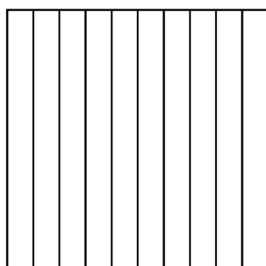
   b. $2\frac{47}{100} = $ ___._____

2. Estimate to locate the points on the number lines.

   a. $2\frac{95}{100}$

   b. $7\frac{52}{100}$

Lesson 6:   Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

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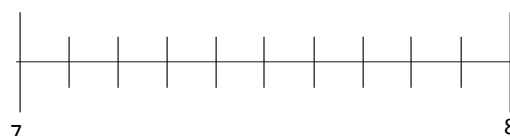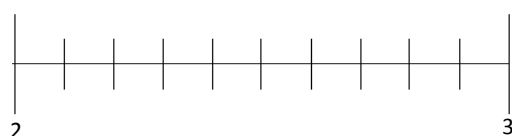
3. Write the equivalent fraction and decimal for each of the following numbers.

a. 1 one 2 hundredths	b. 1 one 17 hundredths
c. 2 ones 8 hundredths	d. 2 ones 27 hundredths
e. 4 ones 58 hundredths	f. 7 ones 70 hundredths

4. Draw lines from dot to dot to match the decimal form to both the unit form and fraction form. All unit forms and fractions have at least one match, and some have more than one match.

7 ones 13 hundredths ●

7 ones 3 hundredths ●

7 ones 3 tenths ●

7 tens 3 ones ●

● 7.30 ●

● 7.3 ●

● 7.03 ●

● 7.13 ●

● 73 ●

● $7\frac{3}{100}$

● 73

● $7\frac{13}{100}$

● $7\frac{30}{100}$

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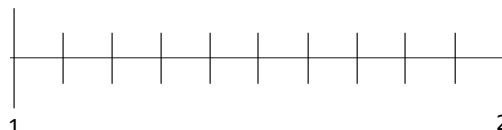
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Name _____   Date _____

1. Estimate to locate the points on the number lines. Mark the point, and label it as a decimal.

   a. $7\frac{20}{100}$

   b. $1\frac{75}{100}$

   7   8

   1   2
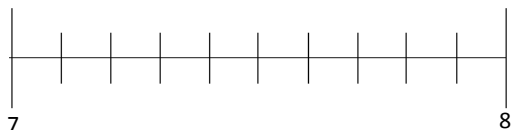
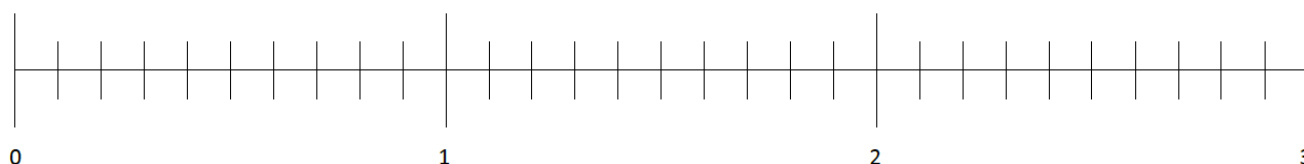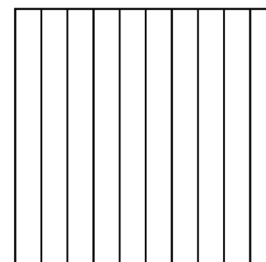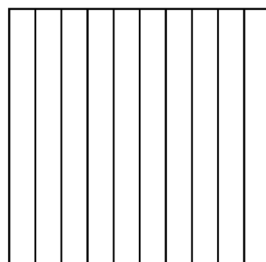2. Write the equivalent fraction and decimal for each number.
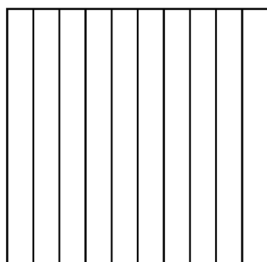
   a. 8 ones 24 hundredths

   b. 2 ones 6 hundredths

Lesson 6: Use the area model and number line to represent mixed numbers with units of ones, tenths, and hundredths in fraction and decimal forms.

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Name _____ Date _____

1. Shade the area models to represent the number, drawing horizontal lines to make hundredths as needed. Locate the corresponding point on the number line. Label with a point, and record the mixed number as a decimal.
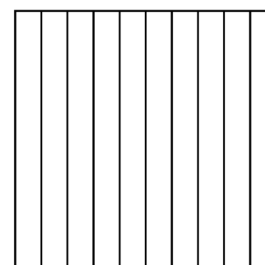
   a. $2\frac{35}{100}$ = ___._____

0   1   2   3

   b. $3\frac{17}{100}$ = ___._____

3   4
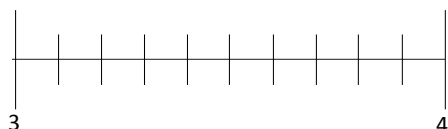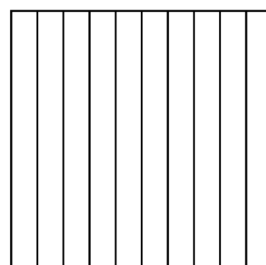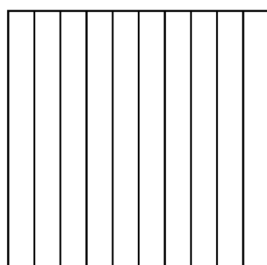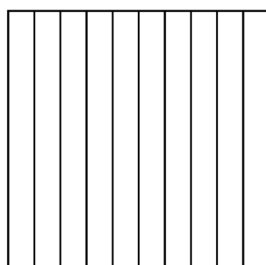
2. Estimate to locate the points on the number lines.

   a. $5\frac{90}{100}$

5   6

   b. $3\frac{25}{100}$

3   4
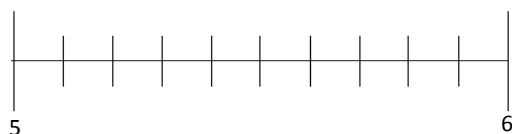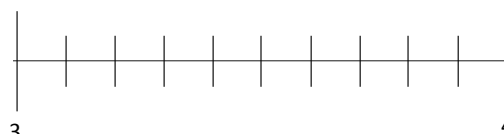
3. Write the equivalent fraction and decimal for each of the following numbers.

a. 2 ones 2 hundredths	b. 2 ones 16 hundredths
c. 3 ones 7 hundredths	d. 1 one 18 hundredths
e. 9 ones 62 hundredths	f. 6 ones 20 hundredths

4. Draw lines from dot to dot to match the decimal form to both the unit form and fraction form. All unit forms and fractions have at least one match, and some have more than one match.

4 ones 18 hundredths ●

4 ones 8 hundredths ●

4 ones 8 tenths ●

4 tens 8 ones ●

● 4.80 ●

● 4.8 ●

● 4.18 ●

● 4.08 ●

● 48 ●

● $4\frac{18}{100}$

● 48

● $4\frac{8}{100}$

● $4\frac{80}{100}$

hundredths area model

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area model

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number line

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Lesson 7

Objective: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

Suggested Lesson Structure

■ Fluency Practice   (11 minutes)
■ Application Problem   (5 minutes)
■ Concept Development   (34 minutes)
■ Student Debrief   (10 minutes)

Total Time   **(60 minutes)**
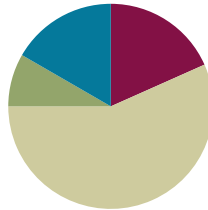
Fluency Practice (11 minutes)

- Count by Hundredths **4.NF.6**   (5 minutes)
- Write the Decimal or Fraction **4.NF.5**   (3 minutes)
- Write the Mixed Number **4.NF.5**   (3 minutes)

Count by Hundredths (5 minutes)

Note: This fluency activity reviews Lessons 4–5.

T: Count by twos to 20, starting at zero.

S: 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20.

T: Count by 2 hundredths to 20 hundredths, starting at 0 hundredths. (Write as students count.)

S: $\frac{0}{100}, \frac{2}{100}, \frac{4}{100}, \frac{6}{100}, \frac{8}{100}, \frac{10}{100}, \frac{12}{100}, \frac{14}{100}, \frac{16}{100}, \frac{18}{100}, \frac{20}{100}$.

$\frac{0}{100}$	$\frac{2}{100}$	$\frac{4}{100}$	$\frac{6}{100}$	$\frac{8}{100}$	$\frac{10}{100}$	$\frac{12}{100}$	$\frac{14}{100}$	$\frac{16}{100}$	$\frac{18}{100}$	$\frac{20}{100}$
$\frac{0}{10}$					$\frac{1}{10}$					$\frac{2}{10}$

T: 1 tenth is the same as how many hundredths?

S: 10 hundredths.

T: (Beneath $\frac{10}{100}$, write $\frac{1}{10}$.)

Continue this process for $\frac{2}{10}$.

T: Let's count by 2 hundredths again. This time, when you come to a tenth, say the tenth. Try not to look at the board.

Lesson 7:   Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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S: $\frac{0}{100}, \frac{2}{100}, \frac{4}{100}, \frac{6}{100}, \frac{8}{100}, \frac{1}{10}, \frac{12}{100}, \frac{14}{100}, \frac{16}{100}, \frac{18}{100}, \frac{2}{10}.$

T: Count backward by 2 hundredths, starting at 2 tenths.

S: $\frac{2}{10}, \frac{18}{100}, \frac{16}{100}, \frac{14}{100}, \frac{12}{100}, \frac{1}{10}, \frac{8}{100}, \frac{6}{100}, \frac{4}{100}, \frac{2}{100}, \frac{0}{100}.$

T: Count by 2 hundredths again. This time, when I raise my hand, stop.

S: $\frac{0}{100}, \frac{2}{100}, \frac{4}{100}, \frac{6}{100}.$

T: (Raise hand.) Say 6 hundredths using digits.

S: Zero point zero six.

T: Continue.

S: $\frac{8}{100}, \frac{1}{10}, \frac{12}{100}, \frac{14}{100}.$

T: (Raise hand.) Say 14 hundredths in digits.

S: Zero point one four.

T: Continue.

S: $\frac{16}{100}, \frac{18}{100}, \frac{2}{10}.$

T: (Raise hand.) Say 2 tenths in digits.

S: Zero point 2.

Write the Decimal or Fraction (3 minutes)

Materials: (T) Hundredths area model (Lesson 6 Fluency Template) (S) Personal white board

Note: This fluency activity reviews Lessons 4–5.

T: (Project hundredths area model. Shade 7 units.) This 1 square is divided into 100 equal parts. Write the fraction of the area that is shaded.

S: (Write $\frac{7}{100}$.)

T: (Write $\frac{7}{100} = $ __.__.) Complete the number sentence.

S: (Write $\frac{7}{100} = 0.07$.)

T: (Project 2 hundredths area models as pictured to the right. Shade one in completely. Shade 7 units in the other area.) Write a fraction to express the area shaded.

S: (Write $1\frac{7}{100}$.)

Lesson 7:   Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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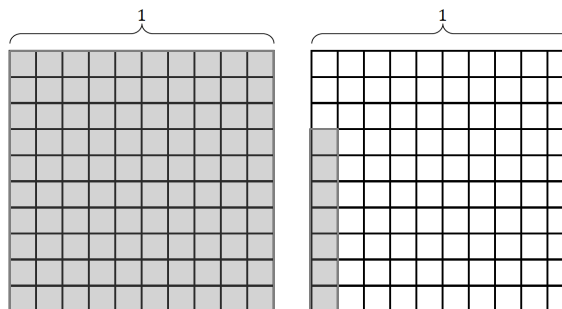
T: (Write $1\frac{7}{100}$ = __.__.) Complete the number sentence.

S: (Write $1\frac{7}{100}$ = 1.07.)

Continue with the following possible sequence: $2\frac{7}{100}$, $\frac{5}{100}$, $1\frac{5}{100}$, $\frac{3}{100}$, and $2\frac{3}{100}$.

T: (Write $3\frac{16}{100}$ = 3 + ── + ── = 3.16.) Complete the number sentence.

S: (Write $3\frac{16}{100}$ = 3 + $\frac{1}{10}$ + $\frac{6}{100}$ = 3.16.)

Continue with the following possible sequence: $2\frac{15}{100}$ and $1\frac{47}{100}$.

Write the Mixed Number (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 6.

T: (Write 1 one 7 hundredths.) Write 1 one 7 hundredths as a mixed number.

S: (Write $1\frac{7}{100}$.)

Continue with the following possible sequence: 1 one 17 hundredths, 3 ones 37 hundredths, 7 ones 64 hundredths, and 9 ones 90 hundredths.

Application Problem (5 minutes)

Materials: (S) Pattern blocks

Use pattern blocks to create at least 1 figure with at least 1 line of symmetry.
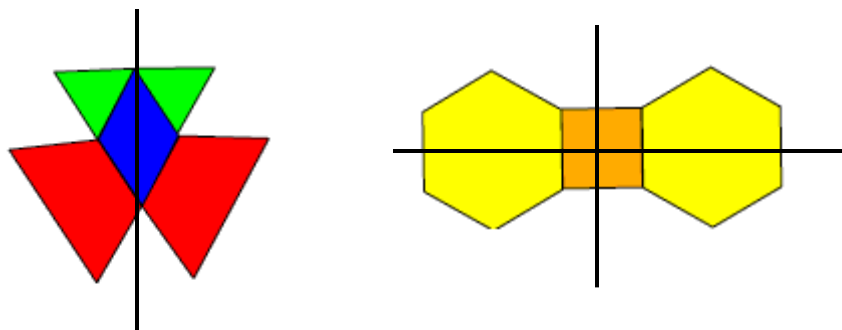
Note: This Application Problem reviews the concept of symmetry (Module 4) to prepare students to explore symmetry in the place value chart in today's Concept Development.

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Lesson 7:   Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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Concept Development (34 minutes)

Materials: (T/S) Place value chart (Template), personal white board

Problem 1: Use place value disks to model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths on the place value chart.

T: (Write 378.73.) Draw place value disks to show 378.73.

S: (Work.)
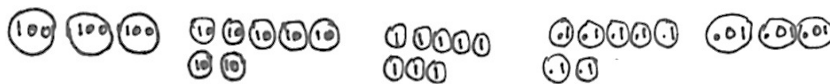
T: Write 378.73 in unit form.

S: (Write 3 hundreds 7 tens 8 ones 7 tenths 3 hundredths.)

3 hundreds   7 tens   8 ones   7 tenths   3 hundredths

T: (Project a place value chart showing hundreds to hundredths, including a decimal point as modeled below.) How is this place value chart different from the charts we have used this year?

S: It has a decimal point and places for tenths and hundredths.

T: Let's show 378.73 on the place value chart. (Distribute the place value chart template, and write 378.73 in the chart.) The digit 3 is written in which places? Tell me the largest place value first.

S: The hundreds and the hundredths.

Hundreds	Tens	Ones	•	Tenths	Hundredths
3	7	8		7	3

T: The digit 7 is written in which places? Tell me the largest place value first.

S: The tens and the tenths.

T: How about the 8?

S: The ones.

Repeat this process with 301.56 and 200.09.

Problem 2: Say the value of each digit.

T: (Show the place value chart with the number 378.73.)   As with any place value chart, the value of each digit is determined by the place value unit.

T: Say the value of the digit in the hundreds place.

S: 3 hundreds.

T: Say the value of the digit in the hundredths place.

S: 3 hundredths.

T: These values sound so much alike. Discuss with your partner how to tell them apart.

S: One is hundreds, and one is hundredths. You have to be careful to say *th*. → One is a whole number, a hundred, and one is a fraction, a hundredth. → It's easier to see how different the values are when you write them as numbers 100 and 0.01. → There are 100 hundredths in one and 100 ones in a hundred. 100 × 100 is 10,000. There are 10,000 hundredths in a hundred.

T: The digit 3 has a greater value in which place?

MP.8 S: The hundreds!

T: Say the value of the digit in the tens place.

S: 7 tens.

T: Say the value of the digit in the tenths place.

S: 7 tenths.

T: These values also sound so much alike. Discuss the difference with your partner.

S: One is tens, and one is tenths. → One is 10, and one is a tenth. → It's easier to see when you write them as numbers: 10 and 0.1.

T: The digit 7 would have a greater value in which place?

S: The tens!

T: Say the value of the 8.

S: 8 ones.

> **NOTES ON**
> **MULTIPLE MEANS**
> **OF ENGAGEMENT:**
>
> Students working above grade level and others may enjoy an independent exploration of symmetry in the place value chart around 1. Ask students to search for patterns in our newly expanded place value chart. Students may find word patterns, such as *tenths* and *tens*, or patterns of ten— multiplying to increase values greater than 1 and dividing to decrease values greater than 1. Students can extend their expression of numbers in expanded form to include their observations of division. This work reaches beyond the scope of Grade 4 standards.

Hundreds	Tens	Ones	•	Tenths	Hundredths
3	7	8		7	3

3 hundreds + 7 tens + 8 ones + 7 tenths + 3 hundredths

Repeat this process with 920.37.

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Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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Problem 3: Express a decimal number in decimal and fraction expanded form.

$$(3 \times 100) + (7 \times 10) + (8 \times 1) + \left(7 \times \frac{1}{10}\right) + \left(3 \times \frac{1}{100}\right) = 378\frac{73}{100}$$

$$(3 \times 100) + (7 \times 10) + (8 \times 1) + (7 \times 0.1) + (3 \times 0.01) = 378.73$$

T: Work with a partner to write 378.73 in expanded form, representing the value of each digit as a multiplication expression.

T: So, some of you expanded it in decimal form (point) and some in fraction form (point). How would you describe to someone what you just did?

S: We took the number apart, one place value at a time. → We decomposed the number by its units. → There are 5 place values and 5 addends. Each addend is an expression that shows the product of the number of units and the size of the unit. → When it came to the tenths and hundredths, you didn't tell us if you wanted decimal form or fraction form, so we could write it either way.

T: In order from largest to smallest, tell me the place value units for this number.

S: Hundreds, tens, ones, tenths, and hundredths.

T: Which digits represent the number of units, in order from left to right?

S: 3, 7, 8, 7, and 3.

T: What do we know about $378\frac{73}{100}$ and 378.73?

S: One is in fraction form, and the other is in decimal form. → They are made of the same 5 units. → They are the same amount. They are just expressed in different forms.

Repeat this process for 340.83 and 456.08. (Point out that when there is a digit of 0 within a number, the digit need not be expressed in expanded form since it adds no value to the number sentence; however, when expressing the number in standard form, the zero is included as a placeholder.)

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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Student Debrief (10 minutes)

Lesson Objective: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How do the place value disks in Problem 1 help to show the value of each digit? How did the unit language help you to write the total value of the place value disks?

- In Problem 2 of the Problem Set, how did the place value chart help to determine the value of each digit?

- Look at the place value charts in Problem 2. *Ten* is found in the word *tenths*, and *hundred* is found in the word *hundredths*. We say that these place values are symmetric. What are they symmetric around? (Note: They are *not* symmetric about the decimal point.) I will shade the ones place to show the symmetry more dramatically.

- In Problem 3, we can write the expanded notation of a number in different ways. What is similar about each of the ways? What is different?

- How did the Application Problem connect to today's lesson?

Symmetry with respect to the ones place

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be aloud to the students.

Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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Name _____   Date _____

1. Write a decimal number sentence to identify the total value of the place value disks.

    a.

    (10) (10)   (0.1)(0.1)(0.1)(0.1)(0.1)   (0.01)(0.01)(0.01)

       2 tens   5 tenths   3 hundredths

    _____ + _____ + _____ = _____

    b.

    (100)(100)(100)(100)(100)   (0.01)(0.01)(0.01)(0.01)

         5 hundreds   4 hundredths

         _____ + _____ = _____

2. Use the place value chart to answer the following questions. Express the value of the digit in unit form.

hundreds	tens	ones	.	tenths	hundredths
4	1	6		8	3

    a. The digit _____ is in the hundreds place. It has a value of _____.

    b. The digit _____ is in the tens place. It has a value of _____.

    c. The digit _____ is in the tenths place. It has a value of _____.

    d. The digit _____ is in the hundredths place. It has a value of _____.

hundreds	tens	ones	.	tenths	hundredths
5	3	2		1	6

    e. The digit _____ is in the hundreds place. It has a value of _____.

    f. The digit _____ is in the tens place. It has a value of _____.

    g. The digit _____ is in the tenths place. It has a value of _____.

    h. The digit _____ is in the hundredths place. It has a value of _____.

Lesson 7:   Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.
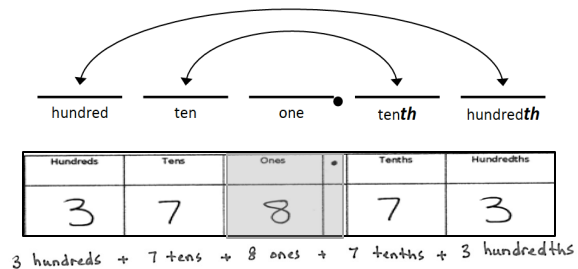
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3. Write each decimal as an equivalent fraction. Then, write each number in expanded form, using both decimal and fraction notation. The first one has been done for you.

Decimal and Fraction Form	Expanded Form	
	Fraction Notation	**Decimal Notation**
$15.43 = 15\frac{43}{100}$	$(1 \times 10) + (5 \times 1) + (4 \times \frac{1}{10}) + (3 \times \frac{1}{100})$ $10 \quad + \quad 5 \quad + \quad \frac{4}{10} \quad + \quad \frac{3}{100}$	$(1 \times 10) + (5 \times 1) + (4 \times 0.1) + (3 \times 0.01)$ $10 \quad + \quad 5 \quad + \quad 0.4 \quad + \quad 0.03$
21.4 = _____		
38.09 = _____		
50.2 = _____		
301.07 = _____		
620.80 = _____		
800.08 = _____		

Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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Name _____   Date _____

1. Use the place value chart to answer the following questions. Express the value of the digit in unit form.

hundreds	tens	ones	.	tenths	hundredths
8	2	7		6	4

   a. The digit _____ is in the hundreds place. It has a value of _____.

   b. The digit _____ is in the tens place. It has a value of _____.

   c. The digit _____ is in the tenths place. It has a value of _____.

   d. The digit _____ is in the hundredths place. It has a value of _____.
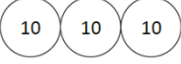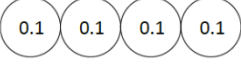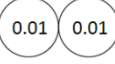
2. Complete the following chart.

Fraction	Expanded Form		Decimal
	Fraction Notation	**Decimal Notation**	
$422\frac{8}{100}$			
	$(3 \times 100) + (9 \times \frac{1}{10}) + (2 \times \frac{1}{100})$		

Lesson 7: Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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Name _____   Date _____

1. Write a decimal number sentence to identify the total value of the place value disks.

    a.
    (10)(10)(10)   (0.1)(0.1)(0.1)(0.1)   (0.01)(0.01)

        3 tens   4 tenths   2 hundredths

        _____ + _____ + _____ = _____

    b.
    (100)(100)(100)(100)(100)   (0.01)(0.01)(0.01)

            4 hundreds   3 hundredths

        _____ + _____ = _____

2. Use the place value chart to answer the following questions. Express the value of the digit in unit form.

hundreds	tens	ones	.	tenths	hundredths
8	2	7		6	4

   a. The digit _____ is in the hundreds place. It has a value of _____.

   b. The digit _____ is in the tens place. It has a value of _____.

   c. The digit _____ is in the tenths place. It has a value of _____.

   d. The digit _____ is in the hundredths place. It has a value of _____.

hundreds	tens	ones	.	tenths	hundredths
3	4	5		1	9

   e. The digit _____ is in the hundreds place. It has a value of _____.

   f. The digit _____ is in the tens place. It has a value of _____.

   g. The digit _____ is in the tenths place. It has a value of _____.

   h. The digit _____ is in the hundredths place. It has a value of _____.

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3. Write each decimal as an equivalent fraction. Then, write each number in expanded form, using both decimal and fraction notation. The first one has been done for you.

Decimal and Fraction Form	Expanded Form	
	Fraction Notation	Decimal Notation
$14.23 = 14\frac{23}{100}$	$(1 \times 10) + (4 \times 1) + (2 \times \frac{1}{10}) + (3 \times \frac{1}{100})$ $10 \quad + \quad 4 \quad + \quad \frac{2}{10} \quad + \quad \frac{3}{100}$	$(1 \times 10) + (4 \times 1) + (2 \times 0.1) + (3 \times 0.01)$ $10 \quad + \quad 4 \quad + \quad 0.2 \quad + \quad 0.03$
25.3 = _____		
39.07 = _____		
40.6 = _____		
208.90 = _____		
510.07 = _____		
900.09 = _____		

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Lesson 7:   Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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hundredths	
tenths	
.	
ones	
tens	
hundreds	

place value chart

Lesson 7:   Model mixed numbers with units of hundreds, tens, ones, tenths, and hundredths in expanded form and on the place value chart.

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Lesson 8

Objective: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
■ Application Problem   (7 minutes)
■ Concept Development   (31 minutes)
■ Student Debrief   (10 minutes)
   Total Time   **(60 minutes)**
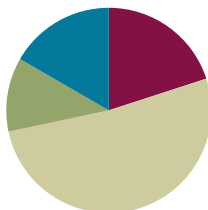
Fluency Practice (12 minutes)

▪ Sprint: Write Fractions and Decimals **4.NF.5**   (9 minutes)
▪ Expanded Form **4.NF.5**   (3 minutes)

Sprint: Write Fractions and Decimals (9 minutes)

Materials: (S) Write Fractions and Decimals Sprint

Note: This Sprint reviews Lessons 4–7.

Expanded Form (3 minutes)

Materials: (T/S) Personal white board

Note: This fluency activity reviews Lesson 7.

T: (Write $4\frac{17}{100}$.) Write 4 and 17 hundredths in expanded fraction form without multiplication.

S: (Write $4\frac{17}{100} = 4 + \frac{1}{10} + \frac{7}{100}$.)

T: Write 4 and 17 hundredths in expanded decimal form.

S: (Write $4.17 = 4 + 0.1 + 0.07$.)

Repeat the process for $25\frac{64}{100}$.

T: (Write 5.93.) Write 5 and 93 hundredths in expanded decimal form.

S: (Write $5.93 = 5 + 0.9 + 0.03$.)

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Lesson 8:   Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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T: Write 5 and 93 hundredths in expanded fraction form.

S: (Write $5\frac{93}{100} = 5 + \frac{9}{10} + \frac{3}{100}$.)

Application Problem (7 minutes)

Jashawn had 5 hundred dollar bills and 6 ten dollar bills in his wallet. Alva had 58 ten dollar bills under her mattress. James had 556 one dollar bills in his piggy bank. They decide to combine their money to buy a computer. Express the total amount of money they have using the following bills:

a. Hundreds, tens, and ones   b. Tens and ones   c. Ones

Note: This Application Problem reviews expanded form and patterns of ten in the place value chart, as taught in Module 1. Reviewing patterns of ten and decomposition of familiar, larger place value units prepares students for today's exploration of decomposition and composition of smaller place value units.

Concept Development (31 minutes)

Materials: (T/S) Area model and place value chart (Template), personal white board

Problem 1: Represent numbers in unit form in terms of different units using the area model.

T: (Place the area model and place value chart template into personal white boards.) Show 2 ones 4 tenths shaded on the area model.

T: (Point to the first rectangle.) How many tenths are in 1?

S: 10 tenths.

T: Record 10 tenths below the first two rectangles. (Point to the third rectangle.) How many tenths are represented?

S: 4 tenths.

T: Record 4 tenths below this rectangle. (Write the addition symbol between the units.) What is 10 tenths plus 10 tenths plus 4 tenths?

S: 24 tenths.

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Lesson 8:   Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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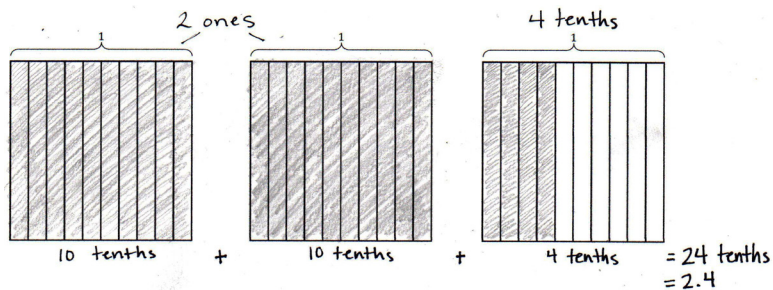
T: (Write 2.4.) So, 2 and 4 tenths is equal to 24 tenths, true?

S: True.

T: Shade 2 ones 40 hundredths on the next set of area models.

T: Record an addition sentence in unit form that tells how many hundredths are shaded.

S: (Write 100 hundredths + 100 hundredths + 40 hundredths = 240 hundredths.)

T: What decimal number is 240 hundredths equal to?

S: 2.40. → 2.4.

T: How can it be equivalent to both?

S: 4 tenths is equal to 40 hundredths, so 0.4 equals 0.40.
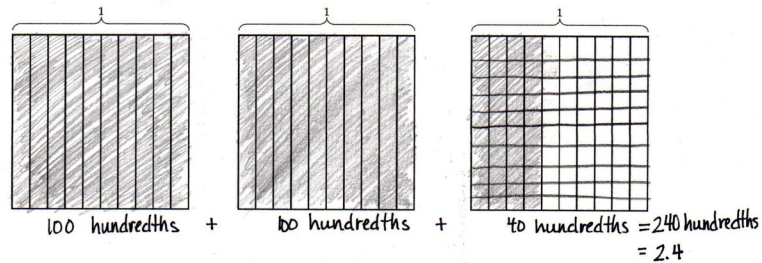
Problem 2: **Represent numbers in unit form in terms of different units using place value disks.**

T: Represent 2 as tenths. How many tenths are in 2 ones?

S: $1 = \frac{10}{10}$, $2 = \frac{10}{10} + \frac{10}{10} = \frac{20}{10}$.

T: Say the equivalence.

S: 2 ones equals 20 tenths.

T: Show 2 ones 4 tenths on your place value chart using place value disks. Express the number in unit form as it is shown on the chart.

S: 2 ones 4 tenths.

T: Decompose the 2 ones, and express them as tenths.

S: 2 ones $= \frac{20}{10}$. There are 20 tenths + 4 tenths = 24 tenths.

T: How can I express 24 tenths as hundredths?

S: You can decompose the tenths to hundredths and count the total number of hundredths. That's too many place value disks to draw!

T: You are right! Let's solve without drawing place value disks. 1 tenth equals how many hundredths?

S: 1 tenth equals 10 hundredths.

T: 2 tenths is equivalent to how many hundredths?

S: 2 tenths equals 20 hundredths.

T: So, 24 tenths equals…? Discuss it with your partner.

S: 240 hundredths. There are 10 times as many hundredths as there are tenths. We showed that using area models. → We can multiply the numerator and denominator by the same number, just like with fractions.

MP.6

T: (Write $\frac{240}{100}$.) Write the equivalent decimal.

S: 2.40 or 2.4.

Repeat with 4.3.

Problem 3: Decompose mixed numbers to express as smaller units.

T: (Write 3.6.) Say this decimal.

S: 3 and 6 tenths.
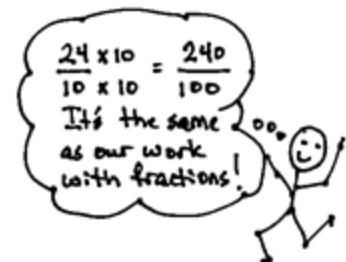
T: How many tenths are in 3 ones?

S: 30 tenths.

T: How many tenths are in 3.6?

S: 36 tenths.

T: In fraction form and unit form, write how many tenths are equal to 3.6.

S: 3.6 = 36 tenths = $\frac{36}{10}$.

T: How many hundredths are in 3 ones?

S: 300 hundredths.

T: How many hundredths are in 6 tenths?

S: 60 hundredths.

T: How many hundredths are in 3.6?

S: 360 hundredths.

T: In fraction form and unit form, write how many hundredths are equal to 3.6.

S: 3.6 = 360 hundredths = $\frac{360}{100}$.

Repeat this process with 5.2 and 12.5.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

> **NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:**
>
> To scaffold the conversion of 24 tenths to 240 hundredths for students working below grade level, offer a few more steps. After verifying that 2 tenths equals 20 hundredths, ask, "5 tenths is equivalent to how many hundredths? (50.) 10 tenths is equivalent to how many hundredths? (100.) 20 tenths is equivalent to how many hundredths? (200.) So, 24 tenths equals…?"

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Lesson 8:   Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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Student Debrief (10 minutes)

Lesson Objective: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Explain why the area model in Problem 1 is a good tool for representing the decimal fraction. How does it help to determine the equivalent decimal number?

- How did drawing the place value disks in Problem 2 help you to understand decomposing from one unit to another?

- How did solving Problem 3 help you to solve Problem 4?

- What strategies did you use when completing the chart in Problem 5? Did you complete one column at a time or one row at a time? Which columns were especially helpful in completing other columns?

- How is decomposing hundreds to tens or tens to ones similar to decomposing ones to tenths or tenths to hundredths?

- When decomposing numbers on the place value chart, each column to the right of another shows 10 times as many parts. Explain why this is so. Even though we have 10 times as many parts, we are really dividing. Explain.

- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A

Number Correct: _____

Write Fractions and Decimals

1.	$\frac{3}{10} =$	.	23.	$2 + \frac{1}{10} + \frac{6}{100} =$	.
2.	$\frac{3}{100} =$	.	24.	$2 + 0.1 + 0.06 =$	.
3.	$\frac{23}{100} =$	.	25.	$3 + 0.1 + 0.06 =$	.
4.	$1\frac{23}{100} =$	.	26.	$3 + 0.1 + 0.04 =$	.
5.	$4\frac{23}{100} =$	.	27.	$3 + 0.5 + 0.04 =$	.
6.	$0.07 =$	—	28.	$2 + 0.3 + 0.08 =$	.
7.	$1.07 =$	—	29.	$2 + 0.08 =$	.
8.	$0.7 =$	—	30.	$1 + 0.3 =$	.
9.	$1.7 =$	—	31.	$10 + 0.3 =$	.
10.	$1.74 =$	—	32.	$1 + 0.4 + 0.06 =$	.
11.	$\frac{4}{100} =$	.	33.	$10 + 0.4 + 0.06 =$	.
12.	$0.6 =$	—	34.	$30 + 0.7 + 0.02 =$	.
13.	$\frac{7}{100} =$	.	35.	$2 + \frac{3}{10} + 0.05 =$	.
14.	$0.02 =$	—	36.	$4 + 0.5 + \frac{3}{100} =$	.
15.	$\frac{9}{100} =$	.	37.	$4 + \frac{3}{100} + 0.5 =$	.
16.	$\frac{10}{100} =$	.	38.	$0.5 + \frac{3}{100} + 4 =$	.
17.	$\frac{10}{100} + \frac{2}{100} =$	.	39.	$20 + 0.8 + 0.01 =$	.
18.	$\frac{1}{10} + \frac{2}{100} =$	.	40.	$4 + \frac{9}{100} + \frac{2}{10} =$	.
19.	$\frac{1}{10} + \frac{3}{100} =$	.	41.	$0.04 + 2 + 0.7 =$	—
20.	$\frac{1}{10} + \frac{4}{100} =$	.	42.	$\frac{6}{10} + 8 + \frac{2}{100} =$	.
21.	$\frac{1}{10} + \frac{9}{100} =$	.	43.	$\frac{5}{100} + 8 + 0.9 =$	—
22.	$3 + \frac{1}{10} + \frac{9}{100} =$	.	44.	$0.9 + 10 + \frac{4}{100} =$	.

Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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B

Number Correct: _____

Improvement: _____

Write Fractions and Decimals

1.	$\frac{1}{10} =$	.	23.	$2 + \frac{1}{10} + \frac{4}{100} =$	.
2.	$\frac{2}{10} =$	.	24.	$2 + 0.1 + 0.04 =$	.
3.	$\frac{3}{10} =$	.	25.	$3 + 0.1 + 0.04 =$	.
4.	$\frac{7}{10} =$	.	26.	$3 + 0.1 + 0.06 =$	.
5.	$\frac{5}{10} =$	.	27.	$3 + 0.5 + 0.06 =$	.
6.	$0.2 =$	—	28.	$2 + 0.4 + 0.09 =$	.
7.	$0.3 =$	—	29.	$2 + 0.06 =$	.
8.	$0.4 =$	—	30.	$1 + 0.5 =$	.
9.	$0.8 =$	—	31.	$10 + 0.5 =$	.
10.	$0.6 =$	—	32.	$1 + 0.2 + 0.04 =$	.
11.	$\frac{4}{10} =$	.	33.	$10 + 0.2 + 0.04 =$	.
12.	$0.9 =$	—	34.	$30 + 0.9 + 0.06 =$	.
13.	$\frac{6}{10} =$	.	35.	$2 + \frac{5}{10} + 0.07 =$	.
14.	$0.5 =$	—	36.	$4 + 0.7 + \frac{5}{100} =$	.
15.	$\frac{9}{10} =$	.	37.	$4 + \frac{5}{100} + 0.7 =$	.
16.	$\frac{10}{10} =$	.	38.	$0.7 + \frac{5}{100} + 4 =$	.
17.	$\frac{11}{10} =$	.	39.	$20 + 0.6 + 0.01 =$	.
18.	$\frac{12}{10} =$	.	40.	$6 + \frac{7}{100} + \frac{4}{10} =$	.
19.	$\frac{17}{10} =$	.	41.	$0.06 + 2 + 0.9 =$	—
20.	$\frac{27}{10} =$	.	42.	$\frac{8}{10} + 6 + \frac{4}{100} =$	.
21.	$\frac{47}{10} =$	.	43.	$\frac{3}{100} + 8 + 0.7 =$	—
22.	$\frac{34}{10} =$	.	44.	$0.7 + 10 + \frac{6}{100} =$	.

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Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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Name _____   Date _____

1. Use the area model to represent $\frac{250}{100}$. Complete the number sentence.

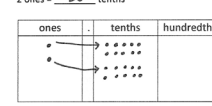   a. $\frac{250}{100}$ = _____ tenths = _____ ones _____ tenths = __.____
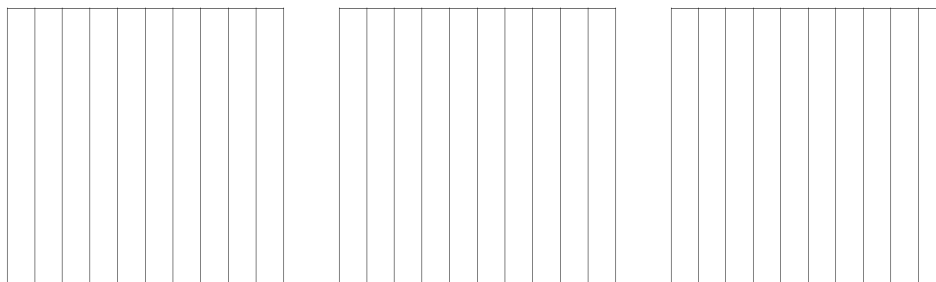
    b. In the space below, explain how you determined your answer to part (a).
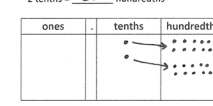
2. Draw place value disks to represent the following decompositions:

    2 ones = _____ tenths

ones	.	tenths	hundredths
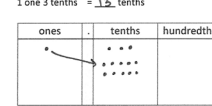
    2 tenths = _____ hundredths
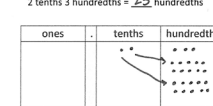
ones	.	tenths	hundredths

    1 one 3 tenths   = _____ tenths

ones	.	tenths	hundredths

    2 tenths 3 hundredths = _____ hundredths

ones	.	tenths	hundredths

3. Decompose the units to represent each number as tenths.

    a.   1 = _____ tenths

    b.   2 = _____ tenths

    c.   1.7 = _____ tenths

    d.   2.9 = _____ tenths

    e.   10.7 = _____ tenths

    f.   20.9 = _____ tenths

4. Decompose the units to represent each number as hundredths.

    a.   1 = _____ hundredths

    b.   2 = _____ hundredths

    c.   1.7 = _____ hundredths

    d.   2.9 = _____ hundredths

    e.   10.7 = _____ hundredths

    f.   20.9 = _____ hundredths

5. Complete the chart. The first one has been done for you.

Decimal	Mixed Number	Tenths	Hundredths
2.1	$2\frac{1}{10}$	21 tenths $\frac{21}{10}$	210 hundredths $\frac{210}{100}$
4.2			
8.4			
10.2			
75.5			

Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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Name _____ Date _____

1. a. Draw place value disks to represent the following decomposition:

    3 ones 2 tenths = _____ tenths

ones	.	tenths	hundredths

    b. 3 ones 2 tenths = _____ hundredths

2. Decompose the units.

    a. 2.6 = _____ tenths

    b. 6.1 = _____ hundredths

![EUREKA MATH]

Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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Name _____   Date _____

1. Use the area model to represent $\frac{220}{100}$. Complete the number sentence.

    a. $\frac{220}{100}$ = _____ tenths = _____ ones _____ tenths = __.____
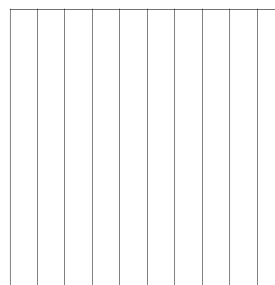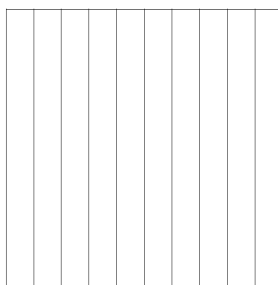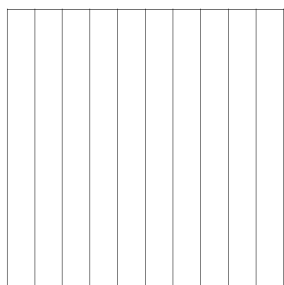
    b. In the space below, explain how you determined your answer to part (a).

2. Draw place value disks to represent the following decompositions:

    3 ones = _____ tenths

ones	.	tenths	hundredths

    3 tenths = _____ hundredths

ones	.	tenths	hundredths

    2 ones 3 tenths = _____ tenths

ones	.	tenths	hundredths

    3 tenths 3 hundredths = _____ hundredths

ones	.	tenths	hundredths

Lesson 8:   Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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3. Decompose the units to represent each number as tenths.

   a. 1 = _____ tenths

   b. 2 = _____ tenths

   c. 1.3 = _____ tenths

   d. 2.6 = _____ tenths

   e. 10.3 = _____ tenths

   f. 20.6 = _____ tenths

4. Decompose the units to represent each number as hundredths.

   a. 1 = _____ hundredths

   b. 2 = _____ hundredths

   c. 1.3 = _____ hundredths

   d. 2.6 = _____ hundredths

   e. 10.3 = _____ hundredths

   f. 20.6 = _____ hundredths

5. Complete the chart. The first one has been done for you.

Decimal	Mixed Number	Tenths	Hundredths
4.1	$4\frac{1}{10}$	41 tenths $\frac{41}{10}$	410 hundredths $\frac{410}{100}$
5.3			
9.7			
10.9			
68.5			

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Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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1   1   1

1   1   1

Tens	Ones	.	Tenths	Hundredths

area model and place value chart

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Lesson 8: Use understanding of fraction equivalence to investigate decimal numbers on the place value chart expressed in different units.

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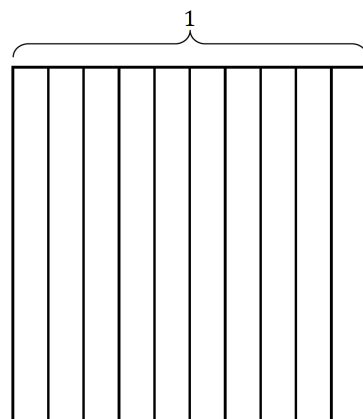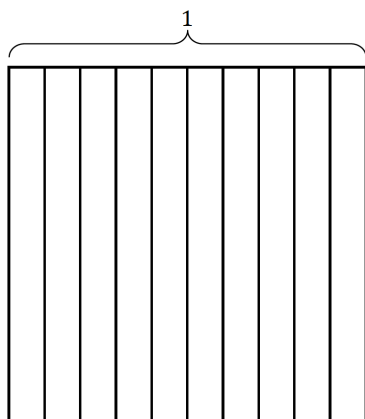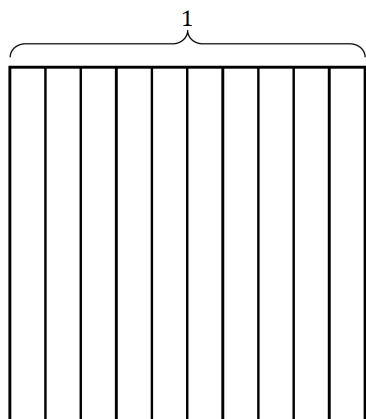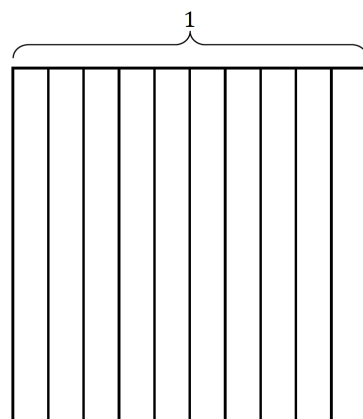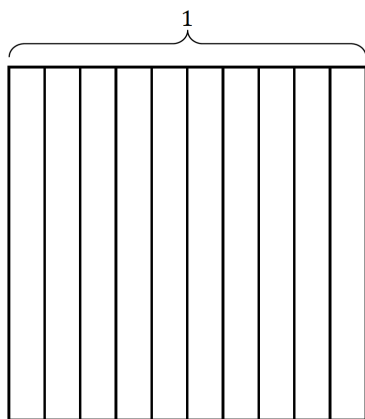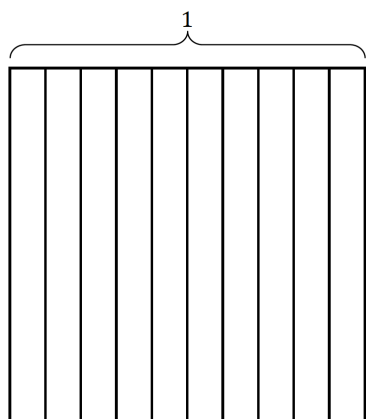
Name _____   Date _____

1. Write the following fractions as equivalent decimals. Then, model each decimal with the given representation.

    a. $\frac{2}{10}$ = _____

    b. $\frac{3}{100}$ = _____

    c. $\frac{4}{10}$ = _____

    d. $\frac{46}{100}$ = _____

    e. $7\frac{6}{10}$ = _____

    7   8

    f. $3\frac{64}{100}$ = _____

    3.6   3.7
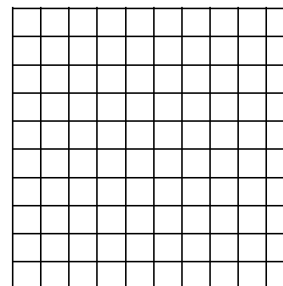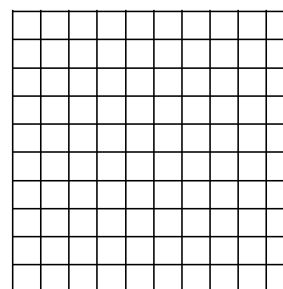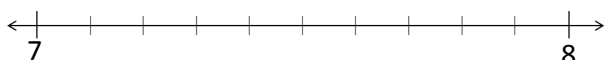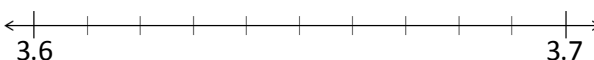
    g. $4\frac{7}{10}$ = _____

ones	.	tenths

    h. $5\frac{72}{100}$ = _____

ones	.	tenths	hundredths

2. Decompose tenths into hundredths using the area model. Express the equivalence of tenths and hundredths with fractions and with decimals.
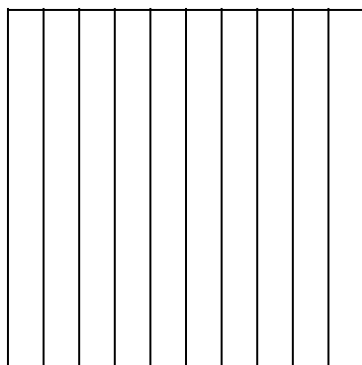
    a. 3 tenths

    b. 1 and 7 tenths
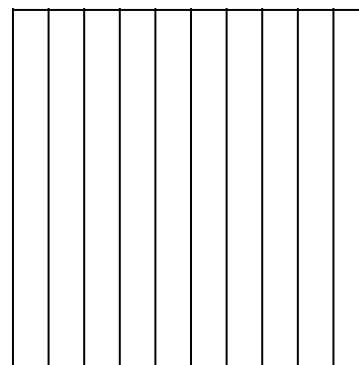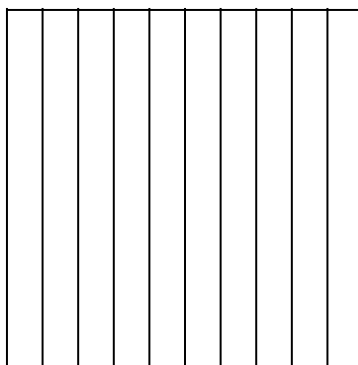
3. Use number bonds to complete parts (a) and (b) below:

    a. Decompose 3.24 by units.

    b. Compose 0.03, 0.5, and 2 as one decimal number.

4. Model the following equivalence on the place value chart using place value disks.

    20 hundredths = 2 tenths

ones	.	tenths	hundredths

5. Complete the following chart.

	Unit Form	Fraction	Fraction Expanded Form	Decimal Expanded Form	Decimal
a.	1 tenth 6 hundredths				
b.		$2\frac{7}{10}$			
c.					6.34
d.				$(1 \times 10) + (6 \times 1) + (5 \times 0.01)$	
e.			$(2 \times 10) + (3 \times 1) + (7 \times \frac{1}{10}) + (8 \times \frac{1}{100})$		

6.   Maya puts groceries into bags. The items and their weights in kilograms are given below.

Bread	Bananas	Cheese	Carrots	Grapes	Eggs
0.25	0.34	0.56	$\dfrac{25}{100}$	$\dfrac{56}{100}$	$\dfrac{34}{100}$

a.   Plot the weight in kilograms of each item on the number line below.

0   1

b.   Write a number sentence using decimals to record the weight in kilograms of the bananas in expanded form.

c.   Write a number sentence using fractions to record the weight in kilograms of the grapes in expanded form.

Maya packs the eggs and cheese into one of the bags. Together, these items weigh $\frac{90}{100}$ kilogram.

d. Use the area model to show that $\frac{90}{100}$ can be renamed as tenths.
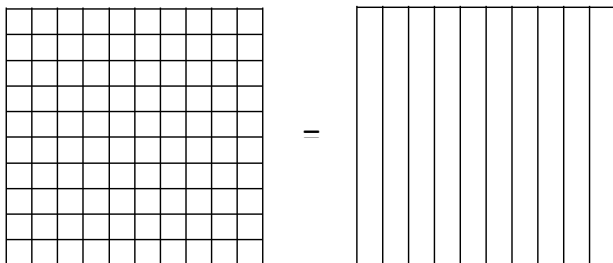
e. Use division to show how $\frac{90}{100}$ can be renamed as tenths.

Maya places the bread into the bag with the eggs and cheese. Together, all three items weigh 1 and 15 hundredths kilograms.

f. Use a model and words to explain how 1 and 15 hundredths can be written as a decimal and as a fraction.

Maya put the rest of the groceries in a second bag. The items in both bags weigh a total of $2\frac{30}{100}$ kilograms.

g. Using a model and words, explain how many tenths are in $2\frac{30}{100}$.

| **Mid-Module Assessment Task** | **Topics A–B** |
| **Standards Addressed** | |

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5   Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)

4.NF.6   Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency.* In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery

Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 **4.NF.6**	Student correctly completes three or fewer parts of the question with little to no modeling.	Student correctly solves at least four parts of the question, providing evidence of some reasoning.	Student correctly solves six or seven of the eight parts of the question. OR Student correctly answers all eight parts but incorrectly models on no more than two parts.	Student correctly writes the equivalent fractions and correctly models using the given representation: a. 0.2 b. 0.03 c. 0.4 d. 0.46 e. 7.6 f. 3.64 g. 4.7 h. 5.72
2 **4.NF.5** **4.NF.6**	Student is unable to correctly answer any of the parts.	Student answers one part correctly.	Student correctly represents the decomposition or correctly writes an equivalent equation in one of the questions. OR Student correctly writes equivalent statements for all parts but incorrectly decomposes in just one part.	Student correctly: ▪ Decomposes the models into hundredths, shading the correct amount. ▪ Expresses the equivalence using fractions and decimals: a. $\frac{3}{10} = \frac{30}{100}$ and $0.3 = 0.30$. b. $1\frac{7}{10} = 1\frac{70}{100}$ and $1.7 = 1.70$.
3 **4.NF.6**	Student is unable to correctly compose or decompose.	Student answers one part correctly.	Student decomposes 3.24 into just two bonds (3, 0.24) and answers part (b) correctly.	Student correctly: a. Decomposes 3.24 into number bonds: 3, 0.2, 0.04. b. Composes 2.53.

A Progression Toward Mastery				
4 **4.NF.5**	Student shows little understanding of the place value disks and equivalence.	Student models equivalence but does not use place value disks.	Student shows some understanding of the place value disks and supplements with a written explanation.	Student correctly uses place value disks to show the equivalence of 20 hundredths and 2 tenths in the place value chart.
5 **4.NF.5** **4.NF.6**	Student correctly answers fewer than 10 of the expressions in the chart.	Student correctly answers 10 to 14 of the expressions in the chart.	Student correctly answers 15 to 19 of the expressions in the chart.	Student correctly answers each expression. (Note: Unit form may have more than one correct answer.) a. 1 tenth 6 hundredths; $\frac{16}{100}$; $(1 \times \frac{1}{10}) + (6 \times \frac{1}{100})$; $(1 \times 0.1) + (6 \times 0.01)$; 0.16. b. 2 ones 7 tenths; $2\frac{7}{10}$; $(2 \times 1) + (7 \times \frac{1}{10})$; $(2 \times 1) + (7 \times 0.1)$; 2.7. c. 6 ones 3 tenths 4 hundredths; $6\frac{34}{100}$; $(6 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100})$; $(6 \times 1) + (3 \times 0.1) + (4 \times 0.01)$; 6.34. d. 1 ten 6 ones 5 hundredths; $16\frac{5}{100}$; $(1 \times 10) + (6 \times 1) + (5 \times \frac{1}{100})$; $(1 \times 10) + (6 \times 1) + (5 \times 0.01)$; 16.05. e. 2 tens 3 ones 7 tenths 8 hundredths; $23\frac{78}{100}$; $(2 \times 10) + (3 \times 1) + (7 \times \frac{1}{10}) + (8 \times \frac{1}{100})$; $(2 \times 10) + (3 \times 1) + (7 \times 0.1) + (8 \times 0.01)$; 23.78.

Module 6:   Decimal Fractions

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A Progression Toward Mastery				
6 **4.NF.5** **4.NF.6**	Student correctly answers fewer than three problems.	Student correctly answers three or four of the seven problems, providing evidence of some reasoning.	Student correctly answers five or six of the seven problems. OR Student answers all parts correctly but without solid evidence or reasoning on fewer than two problems.	Student correctly: a. Plots each item on the number line. b. Responds $0.3 + 0.04 = 0.34$ or $(3 \times 0.1) + (4 \times 0.01) = 0.34$. c. Responds $\frac{5}{10} + \frac{6}{100} = \frac{56}{100}$ or $(5 \times \frac{1}{10}) + (6 \times \frac{1}{100}) = \frac{56}{100}$. d. Represents $\frac{90}{100} = \frac{9}{10}$ in the area models. e. Responds $\frac{90}{100} = \frac{90 \div 10}{100 \div 10} = \frac{9}{10}$. f. Models and explains that 1 and 15 hundredths equals $1\frac{15}{100}$ and 1.15. g. Models and explains that there are 23 tenths in $2\frac{30}{100}$.

Name __Jack__   Date _____

1. Write the following fractions as equivalent decimals. Then, model each decimal with the given representation.

a. $\frac{2}{10} = $ __0.2__
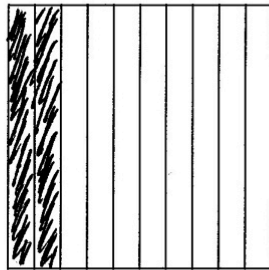
b. $\frac{3}{100} = $ __0.03__
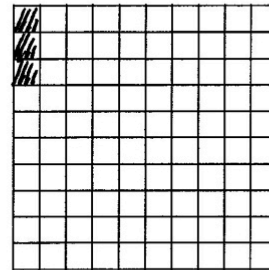
c. $\frac{4}{10} = $ __0.4__

d. $\frac{46}{100} = $ __0.46__
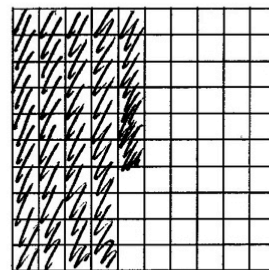
e. $7\frac{6}{10} = $ __7.6__

f. $3\frac{64}{100} = $ __3.64__
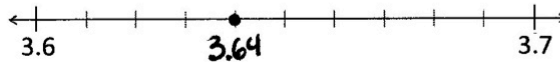
g. $4\frac{7}{10} = $ __4.7__

ones	.	tenths
• • • •		• • • • • • •

h. $5\frac{72}{100} = $ __5.72__

ones	.	tenths	hundredths
• • • • •		• • • • • • •	• •

2. Decompose tenths into hundredths using the area model. Express the equivalence of tenths and hundredths with fractions and with decimals.
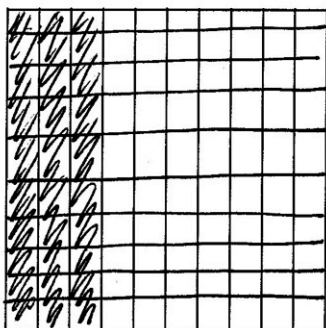
   a. 3 tenths

   $$\frac{3}{10} = \frac{30}{100}$$

   $$0.3 = 0.30$$

   b. 1 and 7 tenths

   $$1\frac{7}{10} = 1\frac{70}{100}$$

   $$1.7 = 1.70$$

3. Use number bonds to complete Parts (a) and (b) below:

   a. Decompose 3.24 by units.

   3.24 → 3, 0.2, 0.04

   b. Compose 0.03, 0.5, and 2 as one decimal number.

   2, 0.5, 0.03 → 2.53

4. Model the following equivalence on the place value chart using place value disks.

20 hundredths = 2 tenths

ones	.	tenths	hundredths

5. Complete the following chart.

|	Unit Form	Fraction	Fraction Expanded Form	Decimal Expanded Form	Decimal
a.	1 tenth 6 hundredths	$\frac{16}{100}$	$(1 \times \frac{1}{10}) + (6 \times \frac{1}{100})$	$(1 \times 0.1) + (6 \times 0.01)$	0.16
b.	2 ones 7 tenths	$2\frac{7}{10}$	$(2 \times 1) + (7 \times \frac{1}{10})$	$(2 \times 1) + (7 \times 0.1)$	2.7
c.	6 ones 3 tenths 4 hundredths	$6\frac{34}{100}$	$(6 \times 1) + (3 \times \frac{1}{10}) + (4 \times \frac{1}{100})$	$(6 \times 1) + (3 \times 0.1) + (4 \times 0.01)$	6.34
d.	1 ten 6 ones 5 hundredths	$16\frac{5}{100}$	$(1 \times 10) + (6 \times 1) + (5 \times \frac{1}{100})$	$(1 \times 10) + (6 \times 1) + (5 \times 0.01)$	16.05
e.	2 tens 3 ones 7 tenths 8 hundredths	$23\frac{78}{100}$	$(2 \times 10) + (3 \times 1) +$ $(7 \times \frac{1}{10}) + (8 \times \frac{1}{100})$	$(2 \times 10) + (3 \times 1) +$ $(7 \times 0.1) + (8 \times 0.01)$	23.78

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Module 6:   Decimal Fractions

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6. Maya puts groceries into bags. The items and their weights in kilograms are given below.

Bread	Bananas	Cheese	Carrots	Grapes	Eggs
0.25	0.34	0.56	$\frac{25}{100}$	$\frac{56}{100}$	$\frac{34}{100}$

a. Plot the weight in kilograms of each item on the number line below.

b. Write a number sentence using decimals to record the weight in kilograms of the bananas in expanded form.

$$0.34 = 0.3 + 0.04$$

c. Write a number sentence using fractions to record the weight in kilograms of the grapes in expanded form.

$$\frac{56}{100} = \frac{5}{10} + \frac{6}{100}$$

Maya packs the eggs and cheese into one of the bags. Together, these items weigh $\frac{90}{100}$ kilogram.

d. Use the area model to show that $\frac{90}{100}$ can be renamed as tenths.
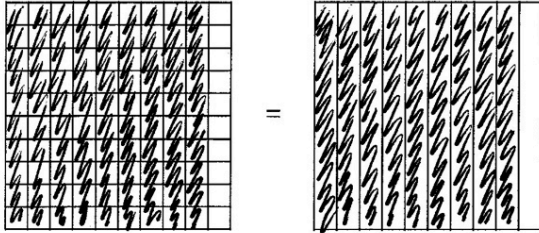
e. Use division to show how $\frac{90}{100}$ can be renamed as tenths.

$$\frac{90}{100} = \frac{90 \div 10}{100 \div 10} = \frac{9}{10}$$

Maya places the bread into the bag with the eggs and cheese. Together, all three items weigh 1 and 15 hundredths kilograms.

f. Use a model and words to explain how 1 and 15 hundredths can be written as a decimal and as a fraction.
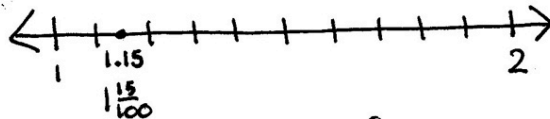
I and 15 hundredths is written as a fraction as $1\frac{15}{100}$ since there is one whole and fifteen hundredths. Written as a decimal, it is 1.15 since there is one whole and fifteen hundredths.
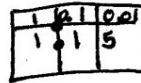
Maya put the rest of the groceries in a second bag. The items in both bags weigh a total of $2\frac{30}{100}$ kilograms.

g. Using a model and words, explain how many tenths are in $2\frac{30}{100}$.

$1 = \frac{10}{10}$ $1 = \frac{10}{10}$ $\frac{30}{100} = \frac{3}{10}$

$\frac{10}{10} + \frac{10}{10} + \frac{3}{10} = \frac{23}{10}$

There are 10 tenths in each whole, so $2 = \frac{10}{10} + \frac{10}{10} = \frac{20}{10}$. $\frac{30}{100} = \frac{3}{10}$ since $\frac{30 \div 10}{100 \div 10} = \frac{3}{10}$. $\rightarrow \frac{20}{10} + \frac{3}{10} = \frac{23}{10}$

There are 23 tenths in $2\frac{30}{100}$.

Topic C
Decimal Comparison

4.NF.7, 4.MD.1, 4.MD.2

Focus Standard:	4.NF.7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
Instructional Days:	3	
Coherence -Links from:	G3–M5	Fractions as Numbers on the Number Line
-Links to:	G5–M1	Place Value and Decimal Fractions

The focus of Topic C is comparison of decimal numbers. In Lesson 9, students compare pairs of decimal numbers representing lengths, masses, or volumes by recording them on the place value chart and reasoning about which measurement is longer than (shorter than, heavier than, lighter than, more than, or less than) the other. Comparing decimals in the context of measurement supports their justifications of their conclusions and begins their work with comparison at a more concrete level.
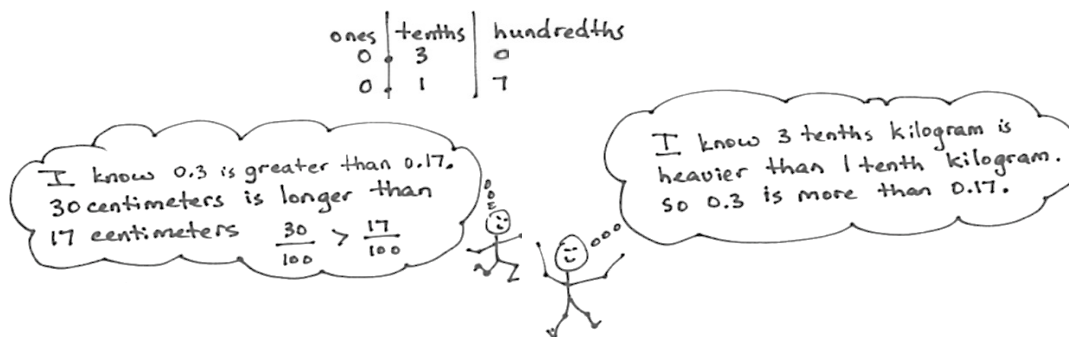
Students move on to more abstract representations in Lesson 10, using area models and the number line to justify their comparison of decimal numbers (**4.NF.7**). They record their observations with the <, >, and = symbols. In both Lessons 9 and 10, the intensive work at the concrete and pictorial levels eradicates the common misconception that occurs, for example, in the comparison of 7 tenths and 27 hundredths, where students believe that 0.7 is less than 0.27 simply because it resembles the comparison of 7 ones and 27 ones. This reinforces the idea that, in any comparison, one must consider the *size of the units.*
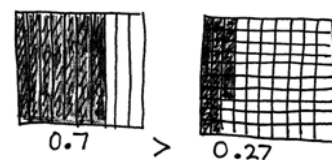
Finally, in Lesson 11, students use their understanding of different ways of expressing equivalent values to arrange a set of decimal fractions in unit, fraction, and decimal form from greatest to least or least to greatest.

$$\frac{37}{100} < 0.5 < 1 \text{ and } 22 \text{ hundredths} < 1\frac{4}{10} < 1.54$$

A Teaching Sequence Toward Mastery of Decimal Comparison
Objective 1: Use the place value chart and metric measurement to compare decimals and answer comparison questions.   **(Lesson 9)**
Objective 2: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.   **(Lesson 10)**
Objective 3: Compare and order mixed numbers in various forms.   **(Lesson 11)**

Lesson 9

Objective: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

Suggested Lesson Structure

■ Fluency Practice   (10 minutes)
■ Application Problem   (5 minutes)
■ Concept Development   (35 minutes)
■ Student Debrief   (10 minutes)

   Total Time   **(60 minutes)**

Fluency Practice (10 minutes)

▪ Decompose Larger Units **4.NF.5**   (3 minutes)

▪ Decimal Fraction Equivalence **4.NF.5**   (5 minutes)

▪ Rename the Decimal **4.NF.5**   (2 minutes)

Decompose Larger Units (3 minutes)

Materials: (S) Personal white board, place value chart (Lesson 7 Template)

Note: This fluency activity reviews Lesson 8.

  T: (Write 1.) Say the number in unit form.

  S: 1 one.

  T: Draw 1 one on your place value chart.

  S: (Draw 1 one disk.)

  T: (Write 1 one = ___ tenths.) Rename 1 one for tenths.

  S: (Cross out the one disk, and draw 10 tenth disks.)

ones	.	tenths	hundredths
		●●●●● ●●●●●	

1 one = 10 tenths

Continue this process using the following possible sequence:

  ▪ Rename 1 one 2 tenths for tenths.

  ▪ Rename 1 tenth for hundredths.

  ▪ Rename 1 tenth 2 hundredths for hundredths.

  ▪ Rename 2 ones 3 tenths for tenths (leads into the next fluency activity).

Lesson 9:   Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Decimal Fraction Equivalence (5 minutes)

Materials: (S) Personal white board, place value chart (Lesson 7 Template)

Note: This fluency activity reviews Lesson 8. For 4 ones 23 hundredths, 1 ten 7 tenths, and 3 tens 4 ones 12 hundredths, have the students express their answers in tenths and hundredths.

T: (Write 2 ones and 3 tenths.) Write the number in digits on your place value chart.

S: (Write the digit 2 in the ones place and the digit 3 in the tenths place.)

T: (Write 2.3 = ___ —.) Write the number as a mixed number.

S: (Write $2.3 = 2\frac{3}{10}$.)

T: (Write $2.3 = 2\frac{3}{10} = \frac{}{10}$.) Write the number as a fraction greater than 1.

S: (Write $2.3 = 2\frac{3}{10} = \frac{23}{10}$.)

ones	.	tenths	hundredths
2	.	3	

$$2.3 = 2\frac{3}{10} = \frac{23}{10}$$

Continue this process for the following possible sequence: 4 ones 23 hundredths, 1 ten 7 tenths, and 3 tens 4 ones 12 hundredths.

Rename the Decimal (2 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 8.

T: (Write 3.1.) Write the decimal as a mixed number.

S: (Write $3\frac{1}{10}$.)

T: (Write $3.1 = 3\frac{1}{10} = \frac{}{10}$.) Complete the number sentence.

S: (Write $3.1 = 3\frac{1}{10} = \frac{31}{10}$.)

T: (Write $3.1 = 3\frac{1}{10} = \frac{31}{10} = \frac{}{100}$.) Complete the number sentence.

S: (Write $3.1 = 3\frac{1}{10} = \frac{31}{10} = \frac{310}{100}$.)

Continue this process for the following possible sequence: 9.8, 10.4, and 64.3.

Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Application Problem (5 minutes)

Kelly's dog weighs 14 kilograms 24 grams. Mary's dog weighs 14 kilograms 205 grams. Hae Jung's dog weighs 4,720 grams.

   a.   Order the weight of the dogs in grams from least to greatest.

   b.   How much more does the heaviest dog weigh than the lightest dog?

Note: This Application Problem reviews decomposition of a number with mixed units. Students need to convert the weight of Kelly's dog to 14,024 grams. The weight of Mary's dog may help them avoid the common error of 1,424 grams because of its inclusion of 205 grams.

Concept Development (35 minutes)

Materials: (T) 2 meter sticks, 2 rolls of different color masking tape (e.g., yellow and blue), metric scale, 4 graduated cylinders, bags of rice, water, food coloring, document camera (S) Personal white board, measurement record (Template)

Materials Note:

- Prepare 2 meter sticks by taping colored masking tape onto the edge of each meter stick to the following lengths: 0.67 m (yellow tape), 0.59 m (blue tape). Do not cover the hash marks or the numbers on the meter sticks.

- Prepare and label 4 bags of rice weighing 0.10 kg (Bag A), 0.65 kg (Bag B), 0.7 kg (Bag C), and 0.46 kg (Bag D).

- Prepare and label four graduated cylinders with water measuring 0.3 liter (Cylinder A), 0.15 liter (Cylinder B), 0.29 liter (Cylinder C), and 0.09 liter (Cylinder D). Use food coloring to help students read the measurements.

> **NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:**
>
> If a document camera, overhead projector, interactive white board, or other means of magnifying the image of the meter stick is not available, consider having students use pre-marked meter sticks at their desks. Certain hardware and home furnishings stores and websites offer meter sticks or tape for free. A meter tape template is also available in Grade 2 Module 2 Lesson 6.

Lesson 9:   Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Problem 1: Compare pairs of decimal numbers representing length.

T: (Hold up the meter stick with the yellow tape that measures 0.67 m, and then place it under the document camera.) Express the length of this yellow tape as a fraction of a meter.

S: $\frac{67}{100}$ meter.

T: On the measurement record, shade the tape diagram to represent the length of the yellow tape on the meter stick. Write the length of the tape in decimal form.

T: (Hold up the meter stick with blue tape that measures 0.59 m, and then project the portion of the meter stick that shows the length of the blue tape under the document camera.) Express the length of this blue tape as a fraction of a meter.

S: $\frac{59}{100}$ meter.

T: On the measurement record, shade the tape diagram to represent the length of the blue tape on the meter stick. Write the length of the tape in decimal form. Record both lengths in a place value chart. (Allow students time to complete the task.)

T: Use the words *longer than* or *shorter than* to compare these two lengths of tape.

S: 0.67 meter is longer than 0.59 meter. → 0.59 meter is shorter than 0.67 meter.
    → 67 centimeters is longer than 59 centimeters, so I know 0.67 meter is longer than 0.59 meter.
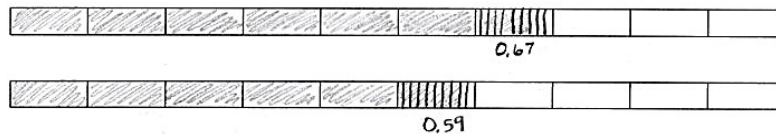
T: Share with a partner. How can the place value chart help you compare these numbers?

S: We can compare the digits in the largest place first. Both measures have 0 in the ones place, so we move to the tenths place. The first tape has 6 tenths. That's greater than 5 tenths. → You don't even need to look at the hundredths place. Once you see that 6 tenths is greater than 5 tenths, you know that the first tape is longer.

Remove enough tape from each meter stick to create the following lengths: 0.4 m and 0.34 m. Repeat the above process.

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Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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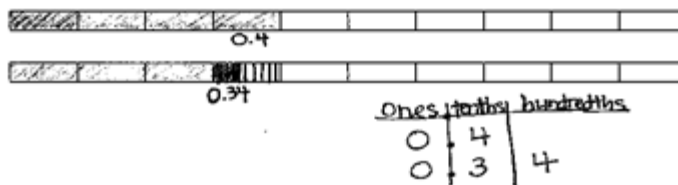
Problem 2: Compare pairs of decimal numbers representing mass.

T: (Place Rice Bag A on the scale.) What is the mass of this bag of rice?

S: Zero point one kilogram. → $\frac{1}{10}$ kilogram. → $\frac{10}{100}$ kilogram (see image below).

T: Record the mass in the table on the measurement record.

Repeat this process for the remaining bags.

T: (Leave Bag D, weighing 0.46 kg, on the scale.) Which bags are heavier than Bag D? How do you know?

S: Bags B and C were heavier than Bag D. → Bag B was 0.65 kg, and Bag C was 0.7 kg. Those numbers are both larger than 0.46 kg, so the bags are heavier. → I looked at my chart, from left to right. In the tenths column, I could see that Bag A was lighter. It had only 1 tenth. Bags B and C were heavier than D because they both had more tenths.

T: Let's look at Bags B and C. Make a statement comparing their mass.

S: 0.65 kilogram is lighter than 0.7 kilogram. → 0.7 kilogram is heavier than 0.65 kilogram.

T: How do you know?

S: I could just see that the bag was fuller and feel that the bag has more mass. → At first, I thought 65 hundredths was more because it looks like you are comparing 65 and 7, and 65 is greater than 7. But then we saw that it was 7 tenths, which is more than 6 tenths. → I realized that 7 tenths is 70 hundredths, and that is greater than 65 hundredths.

T: With your partner, make another statement to compare the bags. You can compare just two items, or you can compare more than two items.

S: (Responses will vary.)

T: Based on these comparisons, what is the mass of the bags in order from heaviest to lightest?

S: 0.7 kg, 0.65 kg, 0.46 kg, 0.1 kg.

T: (Select a student volunteer.) Arrange the bags from heaviest to lightest. Looking at the bags, does it *appear* that we have properly ordered the bags from heaviest to lightest? Do they match the order we determined?

S: Yes.

> **NOTES ON TERMINOLOGY:**
>
> Mass is a fundamental measure of the amount of matter in an object. While weight is a measurement that depends upon the force of gravity (one would weigh less on the moon than one does on Earth), mass does not depend upon the force of gravity. Both words are used here, but it is not important for students to recognize the distinction in mathematics at this time.

Mass of Rice Bags (kilograms)

Rice Bag	ones	.	tenths	hundredths
A	0	.	1	0
B	0	.	6	5
C	0	.	7	
D	0	.	4	6

0.7 kg, 0.65 kg, 0.46 kg, 0.1 kg

Lesson 9:   Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Problem 3: Compare pairs of decimal numbers representing volume.

T: (Place all four graduated cylinders in front of the class.) Express the volume of the liquid in tenths or hundredths liter. (Use the document camera to project the side of Cylinder A so students can see the liter measurements. If this is not possible, select a student to read the volume aloud.)

S: $\frac{3}{10}$ liter. → $\frac{30}{100}$ liter.

T: Record this volume in the table on the measurement record.

Repeat the process for the remaining water samples.

T: If we want to order these samples from least volume to greatest volume, what would the order be? Talk with your partner, and record your thinking on the measurement record. (Circulate to encourage use of the place value chart as students compare the measurements.)

S: (Complete the task.)

S: 0.09 liter, 0.15 liter, 0.29 liter, 0.3 liter.

T: How did you determine the order?

S: The place value chart made it easy to compare the decimals. → We compared the digits in the largest place first. That was the tenths. → In 0.3, there are 3 tenths. That is more than the others. 0.29 comes next, followed by 0.15 and 0.09.

Volume of Liquid (liters)

Cylinder	ones	.	tenths	hundredths
A	0	.	3	
B	0	.	1	5
C	0	.	2	9
D	0	.	0	9

0.09 L, 0.15 L, 0.29 L, 0.3 L

T: (Select a student volunteer to order cylinders from least volume to greatest volume.) Let's look at the cylinders. Do they appear to match the order we determined?

S: Yes!

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Lesson 9:   Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- How do the tape diagrams in Problem 1 support your statements? Make a statement comparing a length from part (a) to a length from part (b).
- Share one of your statements for Problem 2(c). Explain your reasoning.
- How did the place value chart help to compare and order the different measurements in Problem 3?
- How is comparing decimal measurements of length, mass, and volume similar? How is it different?
- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Name _____   Date _____

1. Express the lengths of the shaded parts in decimal form. Write a sentence that compares the two lengths. Use the expression *shorter than* or *longer than* in your sentence.

a.

1 meter

1 meter

b.

1 meter

1 meter

c. List all four lengths from least to greatest.

2. a. Examine the mass of each item as shown below on the 1-kilogram scales. Put an X over the items that are heavier than the avocado.
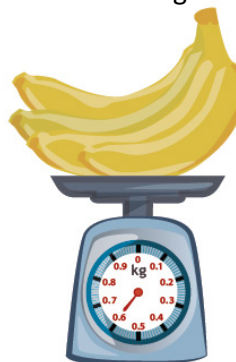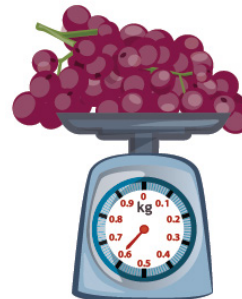
0.2 kg   0.12 kg   0.6 kg   0.61 kg

b. Express the mass of each item on the place value chart.

Mass of Fruit (kilograms)

Fruit	ones	.	tenths	hundredths
avocado				
apple				
bananas				
grapes				

c. Complete the statements below using the words *heavier than* or *lighter than* in your statements.

The avocado is _____ the apple.

The bunch of bananas is _____ the bunch of grapes.

3. Record the volume of water in each graduated cylinder on the place value chart below.
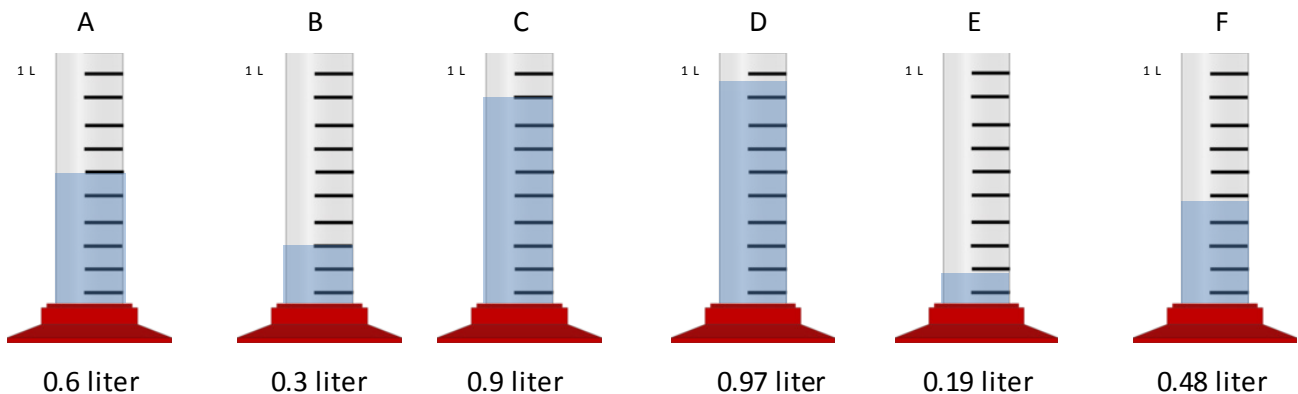
A	B	C	D	E	F
0.6 liter	0.3 liter	0.9 liter	0.97 liter	0.19 liter	0.48 liter

Volume of Water (liters)

Cylinder	ones	.	tenths	hundredths
A				
B				
C				
D				
E				
F				

Compare the values using >, <, or =.

a. 0.9 L _____ 0.6 L

b. 0.48 L _____ 0.6 L

c. 0.3 L _____ 0.19 L

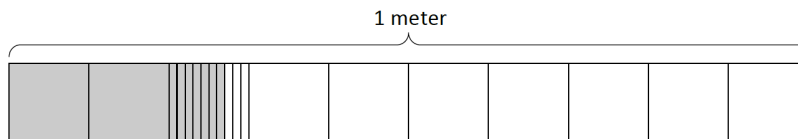d. Write the volume of water in each graduated cylinder in order from least to greatest.

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Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Name _____   Date _____

1. a. Doug measures the lengths of three strings and shades tape diagrams to represent the length of each string as show below. Express, in decimal form, the length of each string.
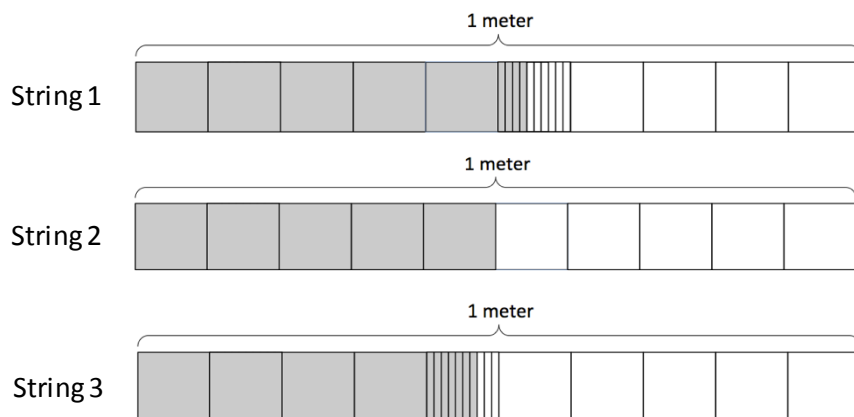
    b. List the lengths of the strings in order from greatest to least.

2. Compare the values below using >, <, or =.

    a. 0.8 kg _____ 0.6 kg

    b. 0.36 kg _____ 0.5 kg

    c. 0.4 kg _____ 0.47 kg

Lesson 9:   Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Name _____   Date _____

1. Express the lengths of the shaded parts in decimal form. Write a sentence that compares the two lengths. Use the expression *shorter than* or *longer than* in your sentence.

   a.

   1 meter

   1 meter

   b.

   1 meter

   1 meter

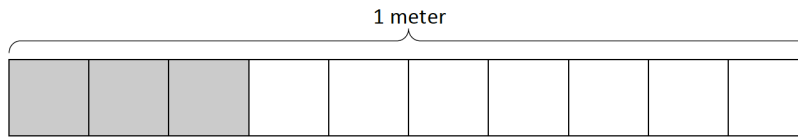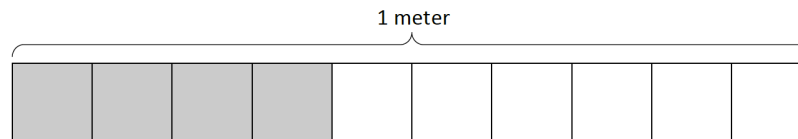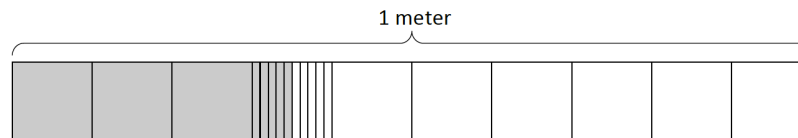   c.   List all four lengths from least to greatest.

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Lesson 9:   Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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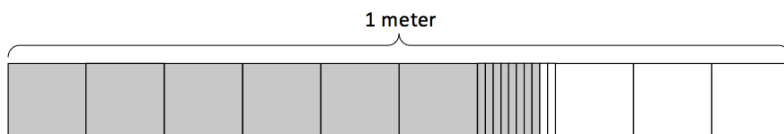
2. a. Examine the mass of each item as shown below on the 1-kilogram scales. Put an X over the items that are heavier than the volleyball

| 0.15 kg | 0.62 kg | 0.43 kg | 0.25 kg |
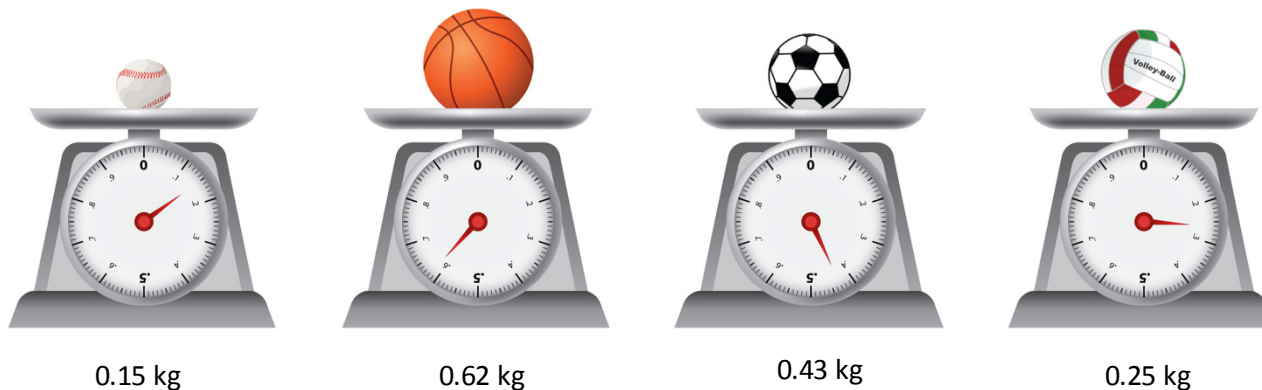
b. Express the mass of each item on the place value chart.

Mass of Sport Balls (kilograms)

Sport Balls	ones	.	tenths	hundredths
baseball				
volleyball				
basketball				
soccer ball				

c. Complete the statements below using the words *heavier than* or *lighter than* in your statements.

The soccer ball is _____ the baseball.

The volleyball is _____ the basketball.

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Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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3. Record the volume of water in each graduated cylinder on the place value chart below.

A	B	C	D	E	F
0.7 liter	0.62 liter	0.28 liter	0.4 liter	0.85 liter	0.2 liter

Volume of Water (liters)

Cylinder	ones	.	tenths	hundredths
A				
B				
C				
D				
E				
F				

Compare the values using >, <, or =.

a.   0.4 L _____ 0.2 L

b.   0.62 L _____ 0.7 L

c.   0.2 L _____ 0.28 L

d.   Write the volume of water in each graduated cylinder in order from least to greatest.

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Lesson 9:   Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Mass of Rice Bags (kilograms)

Rice Bag	ones	.	tenths	hundredths
A				
B				
C				
D				

Volume of Liquid (liters)

Cylinder	ones	.	tenths	hundredths
A				
B				
C				
D				

measurement record

Lesson 9: Use the place value chart and metric measurement to compare decimals and answer comparison questions.

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Lesson 10

Objective: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

Suggested Lesson Structure

■ Fluency Practice   (10 minutes)
■ Application Problem   (5 minutes)
■ Concept Development   (35 minutes)
■ Student Debrief   (10 minutes)
   Total Time   **(60 minutes)**
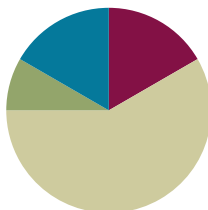
Fluency Practice (10 minutes)

- Decompose Larger Units **4.NF.5**   (3 minutes)
- Decimal Fraction Equivalence **4.NF.5**   (5 minutes)
- Rename the Decimal **4.NF.5**   (2 minutes)

Decompose Larger Units (3 minutes)

Materials: (S) Personal white board, place value chart (Lesson 7 Template)

Note: This fluency activity reviews Lesson 8.

T: (Write 2.) Say the number in unit form.

S: 2 ones.

T: Draw 2 ones on your place value chart.

S: (Draw 2 ones disks.)

T: (Write 2 ones = __ tenths.) Regroup 2 ones for tenths.

S: (Cross out the ones disks, and draw 20 tenths disks. Write 2 ones = 20 tenths.)

2 ones = 20 tenths

Continue with the following possible sequence:

- Regroup 2 ones 5 tenths for tenths.
- Regroup 2 tenths for hundredths.
- Regroup 2 tenths 4 hundredths for hundredths.

Lesson 10:   Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

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Decimal Fraction Equivalence (5 minutes)

Materials: (S) Personal white board, place value chart (Lesson 7 Template)

Note: This fluency activity reviews Lesson 8.

- T: (Write 5 ones 7 tenths.) Write the number in digits on your place value chart.
- S: (Write the digit 5 in the ones place and the digit 7 in the tenths place.)
- T: (Write 5.7 = __ $\frac{}{}$.) Write the number as a mixed number.
- S: (Write 5.7 = $5\frac{7}{10}$.)
- T: (Write 5.7 = $5\frac{7}{10} = \frac{}{10}$.) Write the number as a fraction greater than 1.
- S: (Write 5.7 = $5\frac{7}{10} = \frac{57}{10}$.)
- T: Read this number as written on the chart.
- S: 5 and 7 tenths.
- T: Express the answer as ones and hundredths.
- S: 5 and 70 hundredths.

ones	.	tenths	hundredths
5	.	7	

$$5.7 = 5\frac{7}{10} = \frac{57}{10}$$

Continue with the following possible sequence: 3 ones 8 tenths, 1 ten 9 tenths, and 2 tens 3 ones 3 tenths.

Rename the Decimal (2 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 8.

- T: (Write 5.2.) Write the decimal as a mixed number.
- S: (Write $5\frac{2}{10}$.)
- T: (Write 5.2 = $5\frac{2}{10} = \frac{}{10}$.) Complete the number sentence.
- S: (Write 5.2 = $5\frac{2}{10} = \frac{52}{10}$.)
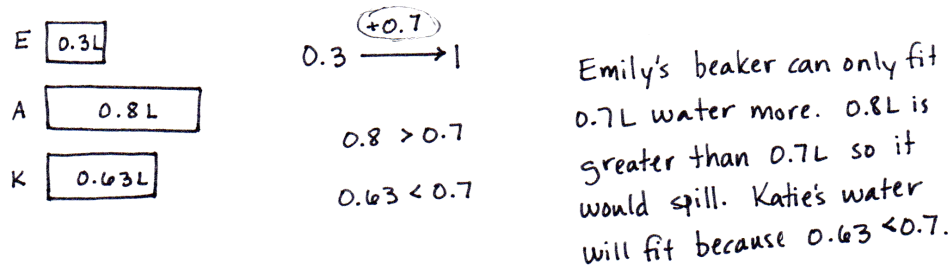- T: (Write 5.2 = $5\frac{2}{10} = \frac{52}{10} = \frac{}{100}$.) Complete the number sentence.
- S: (Write 5.2 = $5\frac{2}{10} = \frac{52}{10} = \frac{520}{100}$.)

Continue with the following possible sequence: 9.6, 10.6, and 78.9.

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Lesson 10: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

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Application Problem (5 minutes)

In science class, Emily's 1-liter beaker contains 0.3 liter of water. Ali's beaker contains 0.8 liter of water, and Katie's beaker contains 0.63 liter of water. Who can pour all of her water into Emily's beaker without going over 1 liter, Ali or Katie?

E [0.3L]

A [0.8 L   ]

K [0.63L]

$0.3 \xrightarrow{+0.7} 1$

$0.8 > 0.7$

$0.63 < 0.7$

Emily's beaker can only fit 0.7L water more. 0.8L is greater than 0.7L so it would still. Katie's water will fit because 0.63 < 0.7.
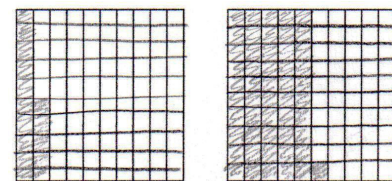
Note: This Application Problem reviews comparison of metric measurements from Lesson 9. Students contextualize and compare volumes of water with measurements of tenths and hundredths. Students may try to use addition and subtraction, but encourage them to use what they know about completing the whole and benchmark numbers.

Concept Development (35 minutes)

Materials: (T/S) Personal white board, comparing with area models (Template), number line
(Lesson 6 Template 2)

Problem 1: Compare pairs of decimal numbers using an area model. Record the comparison using <, >, and =.

T: (Write 0.15 on the board. Distribute the comparing with area models template.) Shade the first area model to represent this decimal.

T: (Write 0.51 on the board.) In the second area model, represent this decimal number.

T: What statements using the phrases *greater than* and *less than* can we make to compare these decimals?

S: 0.51 is greater than 0.15. → 0.15 is less than 0.51.

T: How does the area model help you compare 0.15 and 0.51?

MP.6

S: The shaded part of 0.51 covers a lot more area than the shaded part for 0.15. → I only shaded 1 full column and 1 half of a column to represent 0.15, but I shaded 5 full columns plus another small part of the next column for 0.51, so 0.51 is greater than 0.15. → I have 15 hundredths shaded on the first area model, but I have 51 hundredths shaded on the second area model.

T: (Write <, >, and = on the board.) Use the appropriate comparison symbol to write both statements on comparing with area models.

S: (Write 0.51 > 0.15. 0.15 < 0.51.)

0.15 < 0.51

0.51 > 0.15

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Lesson 10:   Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

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Repeat the process using the following sequence:

- 0.37 and 0.3
- 0.27 and 0.7
- 0.7 and 0.70
- 0.06 and 0.6

Problem 2: Compare decimal numbers on a number line. Record the comparison using <, >, and =.

T: (Distribute the number line template.) Look at the first number line. Label the endpoints as 4 and 3 tenths and 4 and 6 tenths.

T: Label the other tenths that can be labeled on this number line.

S: (Label 4.4 and 4.5.)

T: (Write 4.50 and 4.38 on the board.) Plot and label these two points on the number line.

T: How did you locate the points?

S: I went to 4.5. Since there are no hundredths, you just stop there. → 4.5 is the same as 4.50. → To locate 4.38, I started at 4.3. Then, I went 8 hundredths more to get to 4.38. → I knew 4.38 was 2 hundredths less than 4.4, so I went to 4.4 and counted back 2 hundredths.

T: What statements can we make to compare these decimals?

S: 4.5 is greater than 4.38. → 4.38 is less than 4.5.

T: (Write <, >, and = on the board.) Use the appropriate comparison symbol to write both statements.

S: (Write 4.5 > 4.38. 4.38 < 4.5.)

T: 4.38 has three digits. 4.5 only has two digits. At a quick glance, it appears that 4.38 would have a greater value. Talk with your partner. Why does 4.5 have a greater value even though it has fewer digits?

S: 4.5 has more tenths than 4.38. Tenths are larger than hundredths. → Make the tenths into hundredths. 4 and 5 tenths renamed is 4 and 50 hundredths. Now, it's obvious that it is greater. → Four point five is four point five zero. Now, it has three digits, too. → 4.5 is halfway between 4 and 5, and 4.38 is part of the way between 4 and 4.5, so 4.38 is less than 4.5.
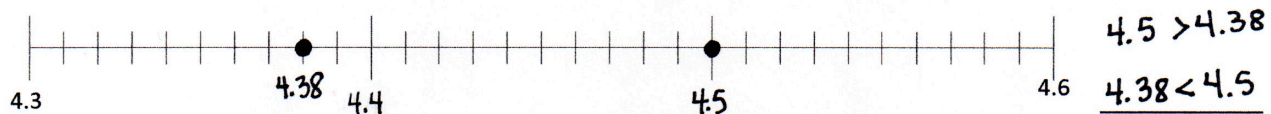
Repeat the process with the number line using the sequence below. Have students label the blank number line to best match each number pair. Ask students to consider what the endpoints should be in order to represent both numbers on the same number line.

- 6.37 _____ 6.3
- 2.68 _____ 2.8
- 10.1 _____ 10.10
- 10.2 _____ 10.02

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Lesson 10: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

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Problem 3: Compare decimal numbers using <, >, and =.

Project the sequence below, and ask students to compare using <, >, and =. With each pair of numbers, ask students to share their reasoning with a partner. They may use the area model, a number line, a place value chart, or other reasonable strategies.

- 6.24 _____ 5.24
- 13.24 _____ 13.42
- 0.48 _____ 2.1
- 2.17 _____ 2.7
- 3.3 _____ 3.30
- 7.9 _____ 7.09
- 8.02 _____ $8\frac{2}{10}$
- 5.3 _____ 5 ones and 3 hundredths
- 5.2 _____ 52 tenths
- 4 ones and 6 tenths _____ 4 ones and 60 hundredths
- 0.25 _____ $\frac{25}{10}$
- $\frac{237}{100}$ _____ 2.73
- 4 tenths _____ 45 hundredths
- 2.31 _____ 23 tenths and 5 hundredths

The sequence above engages students with practice that addresses common misconceptions and becomes increasingly more complex. For instance, the sequence opens with two examples that have the same number of digits and simply requires students to attend to the value of each place. In the next four examples, the pairs being compared have differing numbers of digits. Students come to understand that the value of the number is not dependent on the number of digits. The sequence of the examples then goes on to numbers written in different forms. Students may choose to model the numbers, convert into common units, or rewrite in the same form.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

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Lesson 10:   Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

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Student Debrief (10 minutes)

Lesson Objective: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Compare your area model for Problem 1(d) with your partner's area model. Explain why it was possible to shade both models without decomposing one to hundredths.

- Find an example on your Problem Set where a decimal number with only three digits has a greater value than a decimal number with four digits. Explain why this is so.

- During our lesson, we saw that 0.27 is less than 0.7. Explain why this is so. How can looking at the numbers quickly instead of considering the size of the unit lead to mistakes when comparing? How can we rename 0.7 to compare it easily to 0.27? Which model helped you compare numbers most easily? Was it easier to represent particular problems with certain types of models?

- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Name _____ Date _____

1. Shade the area models below, decomposing tenths as needed, to represent the pairs of decimal numbers. Fill in the blank with <, >, or = to compare the decimal numbers.

   a. 0.23 _____ 0.4

   b. 0.6 _____ 0.38
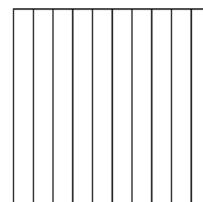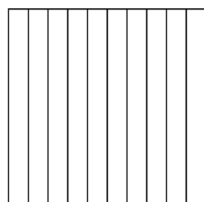
   c. 0.09 _____ 0.9

   d. 0.70 _____ 0.7
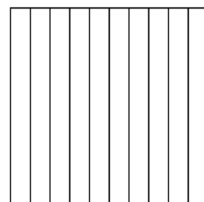
2. Locate and label the points for each of the decimal numbers on the number line. Fill in the blank with <, >, or = to compare the decimal numbers.

   a. 10.03 _____ 10.3

   10.0   10.1   10.2   10.3

   b. 12.68 _____ 12.8

   12.6   12.7   12.8   12.9

Lesson 10: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.
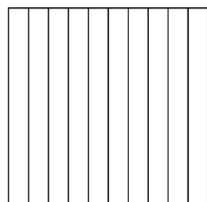
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3. Use the symbols <, >, or = to compare.

   a. 3.42 _____ 3.75

   b. 4.21 _____ 4.12

   c. 2.15 _____ 3.15

   d. 4.04 _____ 6.02

   e. 12.7 _____ 12.70

   f. 1.9 _____ 1.21

4. Use the symbols <, >, or = to compare. Use pictures as needed to solve.

   a. 23 tenths _____ 2.3

   b. 1.04 _____ 1 one and 4 tenths

   c. 6.07 _____ $6\frac{7}{10}$

   d. 0.45 _____ $\frac{45}{10}$

   e. $\frac{127}{100}$ _____ 1.72

   f. 6 tenths _____ 66 hundredths

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Lesson 10: Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

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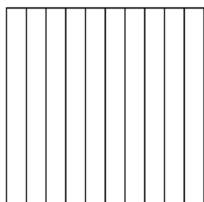Name _____   Date _____

1. Ryan says that 0.6 is less than 0.60 because it has fewer digits. Jessie says that 0.6 is greater than 0.60. Who is right? Why? Use the area models below to help explain your answer.

<center>0.6 _____ 0.60</center>
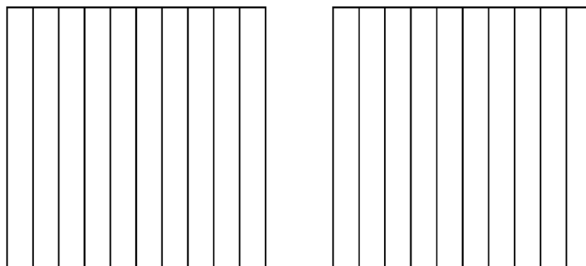
2. Use the symbols <, >, or = to compare.

    a. 3.9 _____ 3.09

    b. 2.4 _____ 2 ones and 4 hundredths

    c. 7.84 _____ 78 tenths and 4 hundredths

Name _____ Date _____

1. Shade the parts of the area models below, decomposing tenths as needed, to represent the pairs of decimal numbers. Fill in the blank with <, >, or = to compare the decimal numbers.
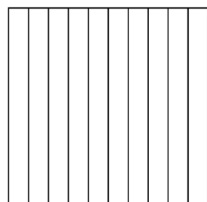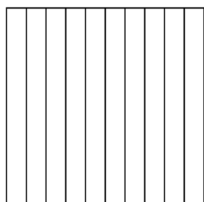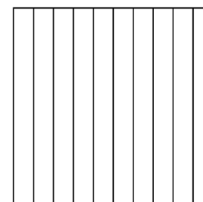
   a.   0.19 _____ 0.3
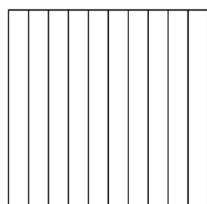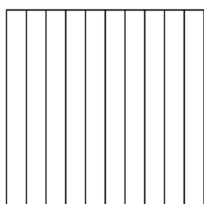
   b.   0.6 _____ 0.06
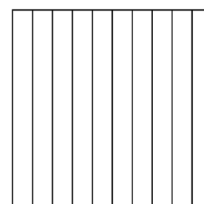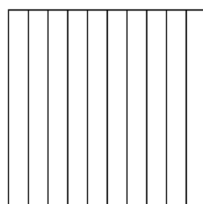
   c.   1.8 _____ 1.53
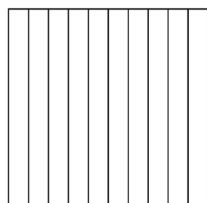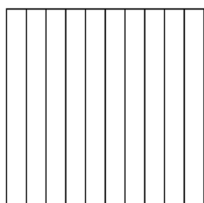
   d.   0.38 _____ 0.7
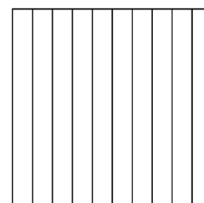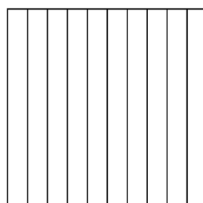
2. Locate and label the points for each of the decimal numbers on the number line. Fill in the blank with <, >, or = to compare the decimal numbers.
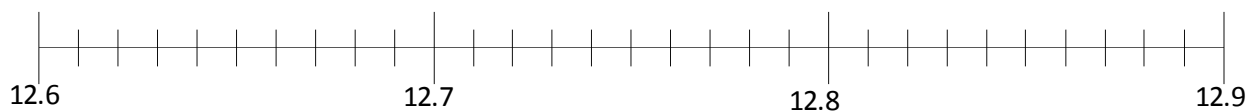
   a.   7.2 _____ 7.02

   |
   7.0   7.1   7.2   7.3

   b.   18.19 _____ 18.3

   |
   18.1   18.2   18.3   18.4

Lesson 10:   Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

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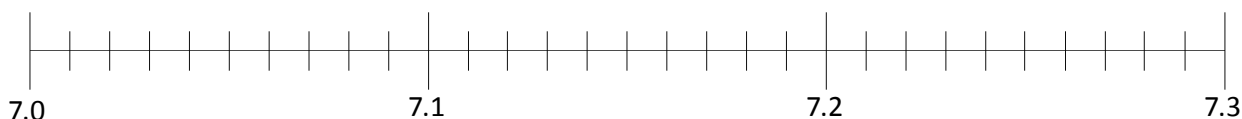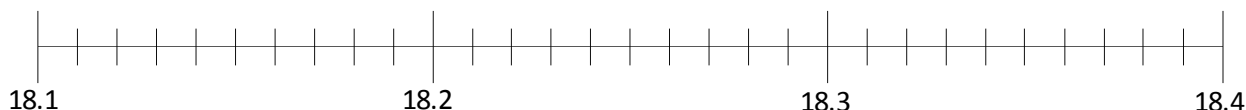
3. Use the symbols <, >, or = to compare.

   a. 2.68 _____ 2.54

   b. 6.37 _____ 6.73

   c. 9.28 _____ 7.28

   d. 3.02 _____ 3.2

   e. 13.1 _____ 13.10

   f. 5.8 _____ 5.92

4. Use the symbols <, >, or = to compare. Use pictures as needed to solve.

   a. 57 tenths _____ 5.7

   b. 6.2 _____ 6 ones and 2 hundredths

   c. 33 tenths _____ 33 hundredths

   d. 8.39 _____ $8\frac{39}{10}$

   e. $\frac{236}{100}$ _____ 2.36

   f. 3 tenths _____ 22 hundredths

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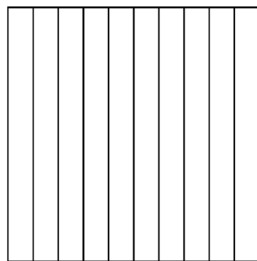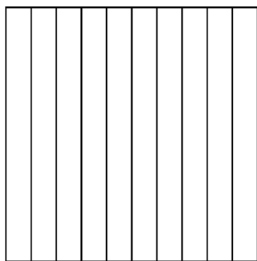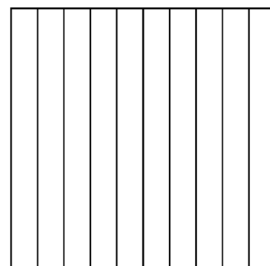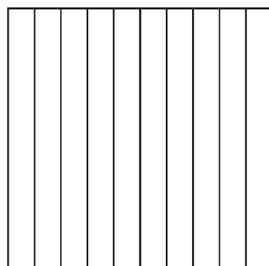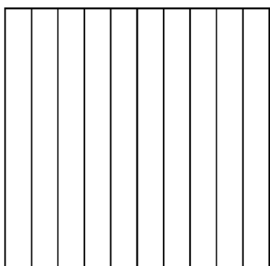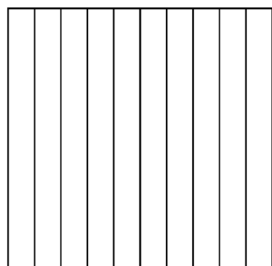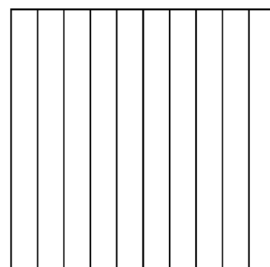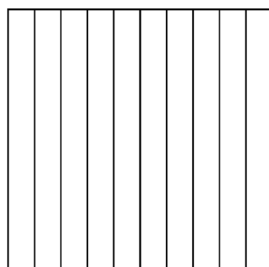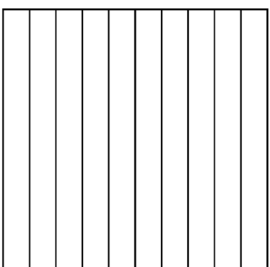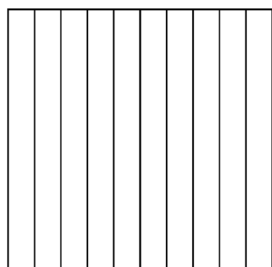
comparing with area models

Lesson 10:   Use area models and the number line to compare decimal numbers, and record comparisons using <, >, and =.

Lesson 11

Objective: Compare and order mixed numbers in various forms.

Suggested Lesson Structure

■ Fluency Practice   (10 minutes)
■ Application Problem   (5 minutes)
■ Concept Development   (35 minutes)
■ Student Debrief   (10 minutes)
   Total Time   **(60 minutes)**
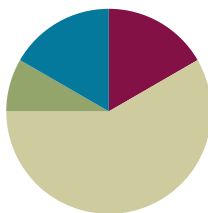
Fluency Practice (10 minutes)

- Expanded Form **4.NBT.2**   (3 minutes)
- Rename the Decimal **4.NF.5**   (4 minutes)
- Compare Decimal Numbers **4.NF.7**   (3 minutes)

Expanded Form (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 7.

   T: (Write $6\frac{13}{100}$.) Write 6 and 13 hundredths in expanded fraction form without multiplication.
   S: (Write $6\frac{13}{100} = 6 + \frac{1}{10} + \frac{3}{100}$.)
   T: Write 6 and 13 hundredths in expanded decimal form.
   S: (Write 6.13 = 6 + 0.1 + 0.03.)

Repeat the process for $54\frac{73}{100}$.

   T: (Write 8.53.) Write 8 and 53 hundredths in expanded decimal form.
   S: (Write 8.53 = 8 + 0.5 + 0.03.)
   T: Write 8 and 53 hundredths in expanded fraction form.
   S: (Write $8\frac{53}{100} = 8 + \frac{5}{10} + \frac{3}{100}$.)

Rename the Decimal (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 8.

> T: (Write 9.4.) Write the decimal as a mixed number.
>
> S: (Write $9\frac{4}{10}$.)
>
> T: (Write $9.4 = 9\frac{4}{10} = \frac{}{10}$.) Complete the number sentence.
>
> S: (Write $9.4 = 9\frac{4}{10} = \frac{94}{10}$.)
>
> T: (Write $9.4 = 9\frac{4}{10} = \frac{94}{10} = \frac{}{100}$.) Complete the
> number sentence.
>
> S: (Write $9.4 = 9\frac{4}{10} = \frac{94}{10} = \frac{940}{100}$.)

Continue with the following possible sequence: 12.3, 4.27,
and 53.8.

> 📋 **NOTES ON
> MULTIPLE MEANS
> OF REPRESENTATION:**
>
> The Compare Decimal Numbers fluency
> activity gives students working below
> grade level and others useful practice
> using the less than (**<**) and greater than
> (**>**) symbols, which are easily confused.
> Mnemonic devices such as imagining
> the **<** symbol to be an alligator mouth
> that eats the larger amount can be
> effective. To enhance the practice, ask
> students to read the comparison
> statements aloud.

Compare Decimal Numbers (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 10.

> T: (Write 2.5 ____ 2.50.) Complete the number sentence, filling in a greater than, less than, or
> equal sign.
>
> S: (Write 2.5 = 2.50.)

Continue with the following possible sequence: 6.74 ____ 6.7, 4.16 ____ 4.61, 3.89 ____ 3.9, 8.64 ____ 8.46,
10.04 ____ 10.4, and 13.28 ____ 13.8.

Application Problem (5 minutes)

While sewing, Kikanza cut 3 strips of colored fabric: a
yellow 2.8-foot strip, an orange 2.08-foot strip, and a
red 2.25-foot strip.

She put the shortest strip away in a drawer and placed
the other 2 strips side by side on a table. Draw a tape
diagram comparing the lengths of the strips on the
table. Which measurement is longer?
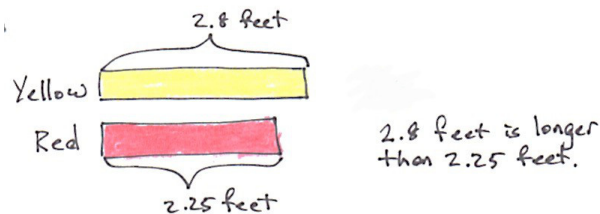
Note: Students apply their comparison skills from Lesson 10 by not including the orange strip in the drawing,
recognizing it is the shortest. This also introduces students to a part–whole tape diagram with decimals
without calculations.

Concept Development (35 minutes)

Materials: (T) Number line (Lesson 6 Template 2) (S) Number line (Lesson 6 Template 2), decimal number flash cards (Template) (1 set per group), personal white board

Note: The onset of Problem 1 asks students to work in small groups. Each group needs one set of flash cards. The recommended group size is three students.

Problem 1: Arrange mixed numbers, fractions, and decimals on a number line.

T: (Distribute 1 set of decimal number flash cards to each group.) In your small groups, work together to arrange your decimal number flash cards in order from least to greatest.

Allow three to five minutes for students to work. Students may renumber the cards if they wish. Do not correct their ordering yet, but do ask students to provide reasoning for their ordering choices.
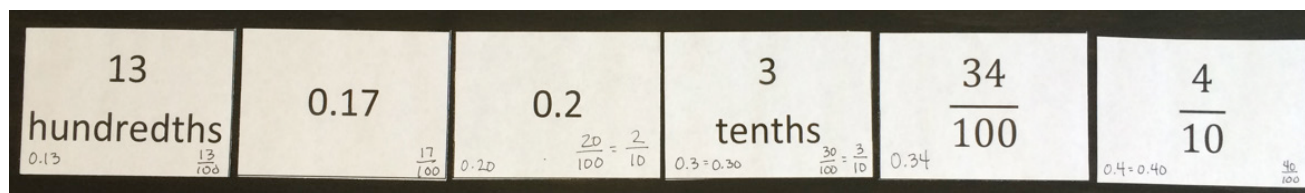
T: We want to plot all of these numbers on the number line. (Distribute the number line template. Project the first number line on the number line template.)

T: What is the smallest number in this set?

S: 13 hundredths.

T: What is the greatest number in this set?

S: 4 tenths.

T: Talk with your group to determine what the most appropriate endpoints are.

S: (Determine the endpoints.)

T: Turn to another group, and compare your endpoints. Discuss how you chose your endpoints.

S: Our endpoints are 1 tenth and 4 tenths since the smallest number in this set is 13 hundredths. We started at the tenth that comes before 13 hundredths.

T: Work with your group to plot and label each number from the set on the number line.

S: (Work with the group to complete the task.)

T: Did your group discover an ordering mistake when it came time to plot the numbers? Explain how you found the mistake.

T: (Project three number lines, completed by students, similar to the ones shown on the following page.) Did these groups represent the numbers using the same form that you did?

S: No, we changed some of the numbers into decimal form so they are all in the same form. → We wrote all the numbers in fraction form. → We left some of them the way they were given to us.

T: Does the form change the order of the numbers?

S: No. No matter which form we used, the numbers are in the same position on the number line.

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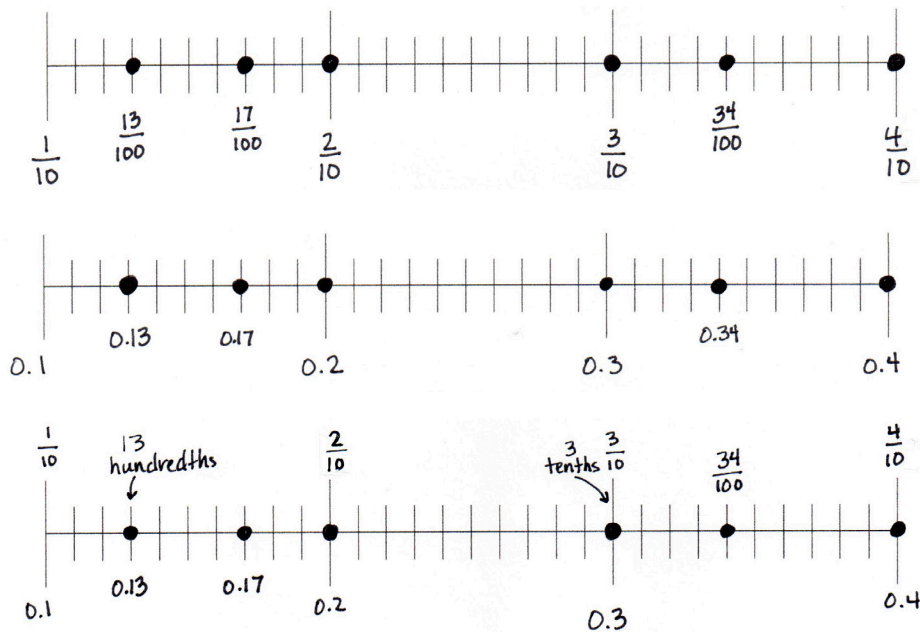
Repeat the process by writing the following sets of numbers:

- 7.92, 8.1, $7\frac{86}{100}$, $\frac{79}{10}$, $\frac{802}{100}$

- $9\frac{5}{10}$, 9.41, $\frac{968}{100}$, $\frac{96}{10}$, 9.7, 9.63

T: Look at your number line. How are your numbers arranged? In what order are they?

S: The numbers go from least to greatest. → The smallest numbers come first. Whenever you read numbers on a number line, they always go in order, with the smallest numbers on the left and larger numbers on the right.

Problem 2: Arrange mixed numbers, fractions, and decimals in order from greatest to least.

T: (Write $\frac{18}{10}$, 1.08, $\frac{18}{100}$, $1\frac{81}{100}$, $\frac{190}{100}$, 1.82.)

T: Instead of using the number line to order the numbers from least to greatest, work with your group to arrange the numbers in order from *greatest to least* using decimal form. Use the > symbol between the numbers as you list them from greatest to least on your personal white board.

S: (Work with the group to complete the task.)

T: List the numbers in order from greatest to least. (Accept numbers in any correct form.)

S: 1.9 > 1.82 > 1.81 > 1.8 > 1.08 > 0.18.

T: How did you decide on the order of the numbers?

S: We changed all of the numbers to decimal form or fraction form because it's easier for us to compare in the same form. → We renamed every number to hundredths. → We left the numbers in tenths and hundredths and used place value to compare: first the ones, then the tenths, and then the hundredths. → We compared the decimals or fractions first. Then, we found where the mixed numbers would go.

Repeat the process with the following sets of numbers:

- $14\frac{5}{10}$, 15.5, $\frac{154}{100}$, 15.05, $14\frac{40}{100}$

- $8\frac{61}{100}$, $8\frac{6}{10}$, $8\frac{1}{10}$, $\frac{816}{100}$, 86, 8.01

Problem 3: Compare and order mixed numbers in the context of a word problem.

T: (Project the following word problem.) During a triple jump contest, Hae Jung jumped 8.76 meters. Marianne jumped $8\frac{7}{10}$ meters. Beth jumped $\frac{880}{100}$ meters. Lily jumped 8.07 meters. In what place did each student rank?

T: Use what you know to answer this question on your personal white board and demonstrate your reasoning. (Allow students time to work.)

MP.4

T: In what place did each student rank?

S: Beth came in first. Hae Jung came in second. Marianne placed third. Lily placed fourth.

T: How did you solve this problem?

S: I changed all of the numbers to decimal form. → I changed all the numbers to fractions. I used hundredths so that they were all the same unit. → I changed everything to a mixed number so I could compare the ones first. I realized I had one fraction with tenths, so I made that 70 hundredths so it would be easier to compare.

Extension: Give six blank flash cards or index cards to each group. Ask the groups to record decimal numbers using various forms for another group to order. Pair up groups, trade cards, and then have the groups check the work of their partnered group.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Compare and order mixed numbers in various forms.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

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Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- In Problem 1(a), which numbers were the easiest for you to plot? Why?

- How did the number line help you to order—or to check the order of—the numbers from least to greatest? Do you think it could be useful to use the number line to order numbers from greatest to least like in Problem 2? Why or why not?

- How could a place value chart help you solve Problem 2(a)? Create an example to share with the class. What other models or tools have we used this year that might help you with Problem 2?

- In Problem 2(b), which numbers did you start ordering first? How did ordering some numbers help you with the remaining numbers? Use specific numbers to explain your process.

- In Problems 3 and 4, how did you make it easier to compare the various numbers? Explain your reasoning.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Lesson 11: Compare and order mixed numbers in various forms.

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Name _____   Date _____

1.   Plot the following points on the number line.

   a. 0.2, $\frac{1}{10}$, 0.33, $\frac{12}{100}$, 0.21, $\frac{32}{100}$

0.1   0.2   0.3   0.4
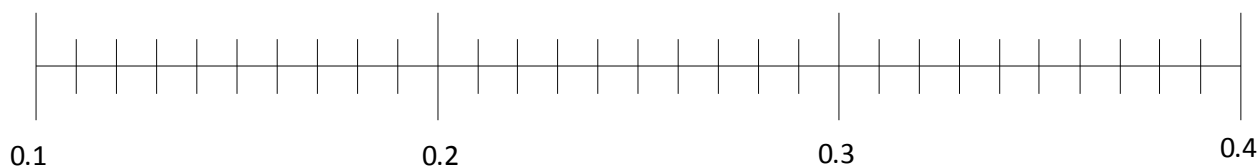
   b. 3.62, 3.7, $3\frac{85}{100}$, $\frac{38}{10}$, $\frac{364}{100}$

3.6   3.7   3.8   3.9
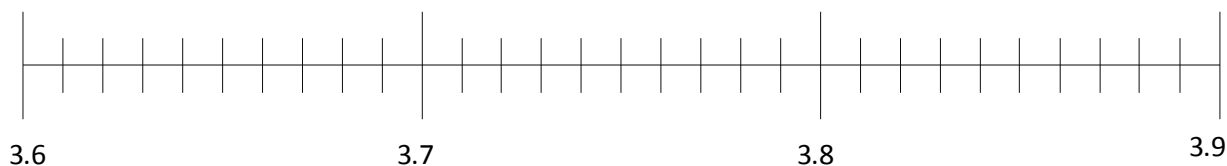
   c. $6\frac{3}{10}$, 6.31, $\frac{628}{100}$, $\frac{62}{10}$, 6.43, 6.40
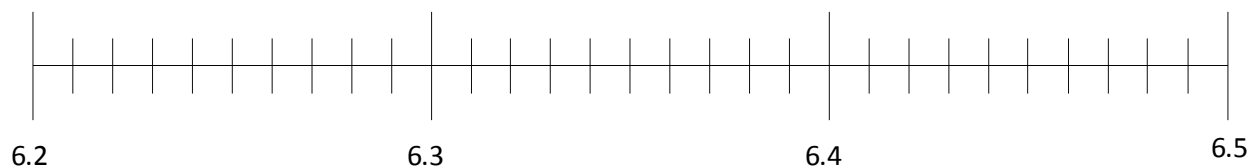
6.2   6.3   6.4   6.5

Lesson 11:   Compare and order mixed numbers in various forms.

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2. Arrange the following numbers in order from greatest to least using decimal form. Use the > symbol between each number.

a. $\frac{27}{10}$, 2.07, $\frac{27}{100}$, $2\frac{71}{100}$, $\frac{227}{100}$, 2.72

_____

b. $12\frac{3}{10}$, 13.2, $\frac{134}{100}$, 13.02, $12\frac{20}{100}$

_____

c. $7\frac{34}{100}$, $7\frac{4}{10}$, $7\frac{3}{10}$, $\frac{750}{100}$, 75, 7.2

_____

3. In the long jump event, Rhonda jumped 1.64 meters. Mary jumped $1\frac{6}{10}$ meters. Kerri jumped $\frac{94}{100}$ meter. Michelle jumped 1.06 meters. Who jumped the farthest?

4. In December, $2\frac{3}{10}$ feet of snow fell. In January, 2.14 feet of snow fell. In February, $2\frac{19}{100}$ feet of snow fell, and in March, $1\frac{1}{10}$ feet of snow fell. During which month did it snow the most? During which month did it snow the least?

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Lesson 11:   Compare and order mixed numbers in various forms.

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Name _____   Date _____

1. Plot the following points on the number line using decimal form.

   1 one and 1 tenth, $\frac{13}{10}$, 1 one and 20 hundredths, $\frac{129}{100}$, 1.11, $\frac{102}{100}$
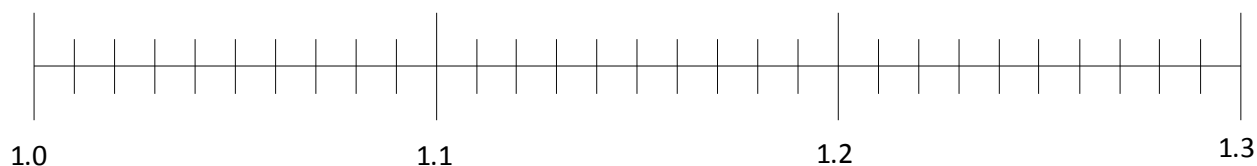
   1.0   1.1   1.2   1.3

2. Arrange the following numbers in order from greatest to least using decimal form. Use the > symbol between each number.

   5.6, $\frac{605}{100}$, 6.15, $6\frac{56}{100}$, $\frac{516}{100}$, 6 ones and 5 tenths

   _____

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Lesson 11:   Compare and order mixed numbers in various forms.

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Name _____   Date _____

1. Plot the following points on the number line using decimal form.

   a.   0.6, $\frac{5}{10}$, 0.76, $\frac{79}{100}$, 0.53, $\frac{67}{100}$

   0.5   0.6   0.7   0.8
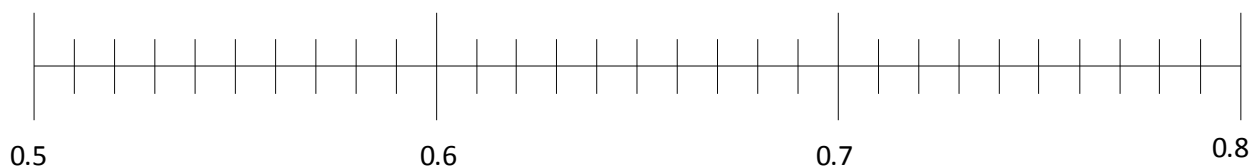
   b.   8 ones and 15 hundredths, $\frac{832}{100}$, $8\frac{27}{100}$, $\frac{82}{10}$, 8.1

   8.1   8.2   8.3   8.4
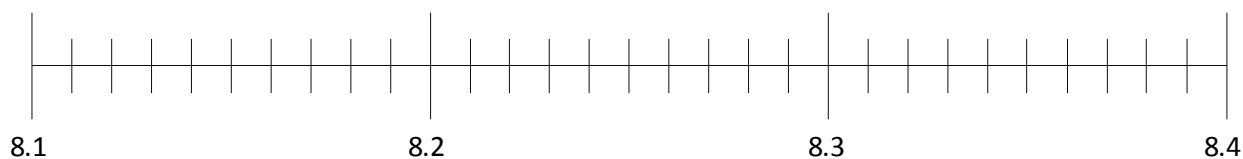
   c. $13\frac{12}{100}$, $\frac{130}{10}$, 13 ones and 3 tenths, 13.21, $13\frac{3}{100}$
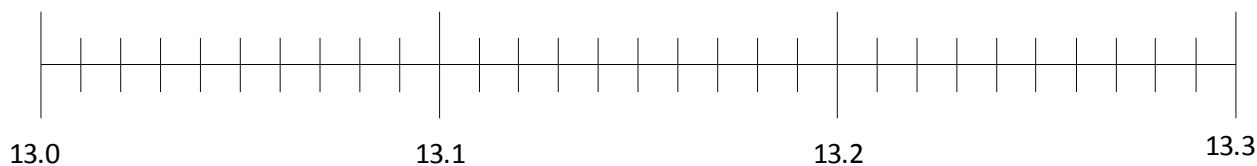
   13.0   13.1   13.2   13.3

2. Arrange the following numbers in order from greatest to least using decimal form. Use the > symbol between each number.

a.   4.03, 4 ones and 33 hundredths, $\frac{34}{100}$, $4\frac{43}{100}$, $\frac{430}{100}$, 4.31

_____

b. $17\frac{5}{10}$, 17.55, $\frac{157}{10}$, 17 ones and 5 hundredths, 15.71, $15\frac{75}{100}$

_____

c.   8 ones and 19 hundredths, $9\frac{8}{10}$, 81, $\frac{809}{100}$, 8.9, $8\frac{1}{10}$

_____

3. In a paper airplane contest, Matt's airplane flew 9.14 meters. Jenna's airplane flew $9\frac{4}{10}$ meters. Ben's airplane flew $\frac{904}{100}$ meters. Leah's airplane flew 9.1 meters. Whose airplane flew the farthest?

4. Becky drank $1\frac{41}{100}$ liters of water on Monday, 1.14 liters on Tuesday, 1.04 liters on Wednesday, $\frac{11}{10}$ liters on Thursday, and $1\frac{40}{100}$ liters on Friday. Which day did Becky drink the most? Which day did Becky drink the least?

3
tenths

0.2

0.17

$\frac{34}{100}$

13
hundredths

$\frac{4}{10}$

decimal number flash cards

Lesson 11: Compare and order mixed numbers in various forms.

Topic D
Addition with Tenths and Hundredths

4.NF.5, 4.NF.6, 4.NF.3c, 4.MD.1

Focus Standards:	4.NF.5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)
	4.NF.6	Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
Instructional Days:	3	
Coherence -Links from:	G3–M5	Fractions as Numbers on the Number Line
-Links to:	G5–M2	Multi-Digit Whole Number and Decimal Fraction Operations

Topic D brings together students' work with addition of fractions and their work with decimals. In Lesson 12, students begin at the pictorial level, decomposing tenths using the area model and place value chart to add tenths and hundredths. They progress to using multiplication to generate equivalent fractions and express the sum in fraction form as a decimal, as pictured below.
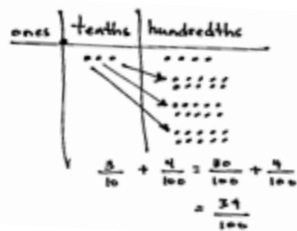
$$\frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100}$$

$$\frac{3}{10} + \frac{4}{100} = \frac{30}{100} + \frac{4}{100} = \frac{34}{100} = 0.34$$

Students next apply what they know about fraction addition to use multiple strategies to solve sums of tenths and hundredths with totals greater than 1 (see the two examples pictured below), again expressing the solution in decimal form.

In Lesson 13, students add ones, tenths, and hundredths in decimal form by converting the addends to mixed numbers in fraction form, creating like denominators, and applying their understanding of the addition of mixed numbers. Once the decimal fractions are added (**4.NF.5**), the number sentence is written in decimal notation (**4.NF.6**).

$$5.6 + 4.53 = 5\frac{6}{10} + 4\frac{53}{100}$$
$$= 5\frac{60}{100} + 4\frac{53}{100}$$
$$= 9\frac{60}{100} + \frac{53}{100}$$
$$= 9\frac{113}{100}$$
$$= 10\frac{13}{100}$$
$$5.6 + 4.53 = 10.13$$

$$5.6 + 4.53 = 5\frac{6}{10} + 4\frac{53}{100}$$
(branch: $\frac{4}{10}$, $\frac{13}{100}$)
$$= 9 + 1 + \frac{13}{100}$$
$$= 10\frac{13}{100}$$
$$5.6 + 4.53 = 10.13$$

$$5.6 + 4.53 = 5\frac{60}{100} + 4\frac{53}{100}$$
(branch: $\frac{40}{100}$, $\frac{13}{100}$)
$$= 10\frac{13}{100}$$
$$5.6 + 4.53 = 10.13$$

The addition of decimals is a Grade 5 standard. By converting addends in decimal form to fraction form, Grade 4 students strengthen their understanding both of fraction and decimal equivalence and of fraction addition.

In Lesson 14, students apply this work to solve measurement word problems involving addition. They convert decimals to fraction form, solve the problem, and write their statement using the decimal form of the solution as pictured below.

An apple orchard sold 140.5 kilograms of apples in the morning. The orchard sold 15.85 kilograms more apples in the afternoon than in the morning. How many total kilograms of apples were sold that day?

M [140.5 kg]
A [140.5 kg | 15.85kg]   } ?
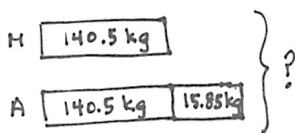
Solution A
$$140\frac{5}{10} + 15\frac{85}{100} = 155\frac{50}{100} + \frac{85}{100}$$
$$= 155\frac{135}{100}$$
$$= 156\frac{35}{100}$$
$$140\frac{5}{10} + 156\frac{35}{100} = 296\frac{50}{100} + \frac{35}{100}$$
$$= 296\frac{85}{100}$$

The apple orchard sold 296.85 kilograms of apples.

Solution B
$$\left(2 \times 140\frac{5}{10}\right) + 15.85 = 280\frac{10}{10} + 15\frac{85}{100}$$
$$= 296\frac{85}{100}$$

The apple orchard sold 296.85 kilograms that day.

Topic D: Addition with Tenths and Hundredths

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A Teaching Sequence Toward Mastery of Addition with Tenths and Hundredths

Objective 1: Apply understanding of fraction equivalence to add tenths and hundredths.
(Lesson 12)

Objective 2: Add decimal numbers by converting to fraction form.
(Lesson 13)

Objective 3: Solve word problems involving the addition of measurements in decimal form.
(Lesson 14)

Lesson 12

Objective: Apply understanding of fraction equivalence to add tenths and hundredths.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
■ Application Problem   (5 minutes)
■ Concept Development (33 minutes)
■ Student Debrief   (10 minutes)
   Total Time   **(60 minutes)**
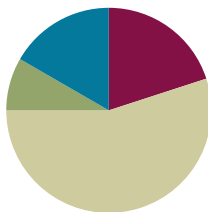
Fluency Practice (12 minutes)

▪ Add and Subtract **4.NBT.4**   (3 minutes)
▪ Compare Decimal Numbers **4.NF.7**   (4 minutes)
▪ Order Decimal Numbers **4.NF.7**   (5 minutes)

Add and Subtract (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews adding and subtracting using the standard algorithm.

   T: (Write 473 thousands 379 ones + 473 thousands 379 ones.) On your personal white board, write this addition sentence in standard form.

   S: (Write 473,379 + 473,379.)

   T: Add using the standard algorithm.

   S: (Write 473,379 + 473,379 = 946,758 using the standard algorithm.)

Continue the process for 384,917 + 384,917.

   T: (Write 700 thousand 1 ten.) On your board, write this number in standard form.

   S: (Write 700,010.)

> **NOTES ON MULTIPLE MEANS OF EXPRESSION:**
>
> Challenge students working above grade level and others to apply efficient alternative strategies learned since Grade 1 to solve the Add and Subtract fluency activity. For example, students can avoid renaming to solve 700,010 − 199,856 by subtracting 11 from both the minuend and the subtrahend (i.e., 699,999 − 199,845) or by adding 144 to both the minuend and subtrahend (i.e., 700,154 − 200,000). Prompt students to explore and explain why the difference is the same using all three methods.

T: (Write 199 thousands 856 ones.) Subtract this number from $700,010$ using the standard algorithm.

S: (Write $700,010 - 199,856 = 500,154$ using the standard algorithm.)

Continue the process for $900,080 - 288,099$.

Compare Decimal Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 10.

T: (Write 3.20 __ 3.2.) Complete the number sentence, filling in a greater than, less than, or equal sign.

S: (Write 3.20 = 3.2.)

Continue with the following possible sequence: 7.8 __ 7.85, 5.72 __ 5.27, 2.9 __ 2.89, 6.24 __ 6.42, 10.8 __ 10.08, and 14.39 __ 14.9.

Order Decimal Numbers (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 11.

T: (Write 0.3, $\frac{1}{10}$, and 0.44.) Arrange the numbers in order from least to greatest.

S: (Write $\frac{1}{10}$, 0.3, and 0.44.)

Continue with the following possible sequence:

- $\frac{13}{10}$, $\frac{43}{100}$, $\frac{1}{100}$, 0.54, 0.1, 0.55, 0.66
- $\frac{93}{10}$, 3 ones and 9 hundredths, $\frac{39}{100}$, 30 and 9 tenths, $\frac{390}{100}$, 3.93

Application Problem (5 minutes)

On Monday, $1\frac{7}{8}$ inches of rain fell. On Tuesday, it raised $\frac{1}{4}$ inch. What was the total rainfall for the two days?

Note: This Application Problem builds from Module 5 work where students learned to add fractions with related units (wherein one denominator is a factor of the other) and mixed numbers. Review of this lesson leads to today's Concept Development where students convert tenths to hundredths before adding decimal numbers.
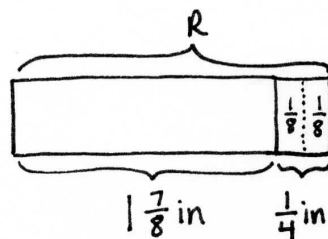
$R = 1\frac{7}{8}$ in $+ \frac{1}{4}$ in $\frac{1}{4} = \frac{1 \times 2}{4 \times 2} = \frac{2}{8}$

$R = 2$ in $+ \frac{1}{8}$ in   The total rainfall for the 2 days was $2\frac{1}{8}$ inches.

$R = 2\frac{1}{8}$ in

Lesson 12: Apply understanding of fraction equivalence to add tenths and hundredths.

engage^ny   **185**

Concept Development (33 minutes)

Materials: (T) Area model and place value chart (Template) (S) Personal white board, area model and place value chart (Template)

Problem 1: Add tenths and hundredths written in unit form using pictorial models.

T: What is 3 girls + 2 girls?

S: 5 girls.

T: What is 3 girls + 2 students?

S: We can't add girls and students. The units don't match.

T: True. But, let's say the girls are students. Tell me the new number sentence, renaming to make like units.

S: 3 students + 2 students = 5 students.

T: What is 3 fourths + 2 fourths?

S: 5 fourths.

T: What is 3 fourths + 1 half? How can you solve? Discuss with your partner.

S: We have to make like units. → We have to rename a half as fourths. → We can convert halves to fourths: $\frac{1}{2} = \frac{2}{4}$. Then, we can add, $\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$.

T: Is this true? (Write $\frac{3}{4} + \frac{1}{2} = \frac{3}{4} + \frac{2}{4}$.)

S: Yes!

T: 3 tenths + 4 tenths is…?

S: 7 tenths.

T: 3 tenths + 4 hundredths is…? How can you solve? Discuss with a partner.

S: We have to make like units. → We have to rename 3 tenths as 30 hundredths. → We can decompose tenths to hundredths. → We can convert tenths to hundredths: $\frac{3}{10} = \frac{30}{100}$. Then, we can add, $\frac{30}{100} + \frac{4}{100} = \frac{34}{100}$.

T: Is this true? (Write 3 tenths + 4 hundredths = 30 hundredths + 4 hundredths.)

S: Yes!

T: Model the addition using an area model or place value chart. Show the conversion of tenths to hundredths. Discuss with your partner.

S: I drew the area model showing 3 tenths and 4 hundredths. Then, I decomposed the area into hundredths to make like units. That meant that I had 30 hundredths and 4 hundredths to have a total of 34 hundredths. → On the place value chart, I drew 3 tenths and 4 hundredths and then decomposed each tenth into 10 hundredths. That gave me a total of 34 hundredths.
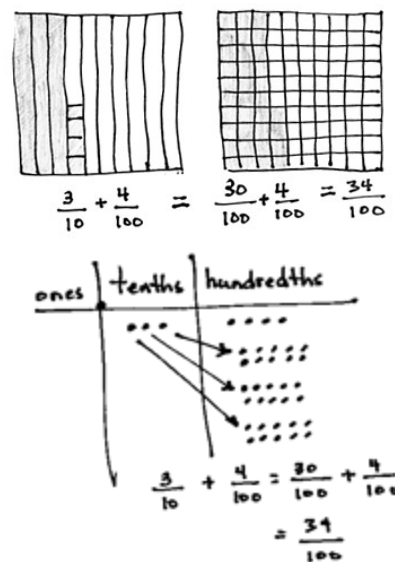
Repeat the process for 2 tenths + 17 hundredths and 36 hundredths + 6 tenths.

Lesson 12: Apply understanding of fraction equivalence to add tenths and hundredths.

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Problem 2: Add tenths and hundredths by converting using multiplication. Express the sum as a decimal.

T: (Write $\frac{3}{10} + \frac{13}{100}$.) Are we ready to add?

S: No.

T: Discuss with a partner. How can we solve using multiplication to make like units?

S: Multiply both the numerator and denominator of 3 tenths by 10 so that we have like units—hundredths.
→ Convert 3 tenths to hundredths. $\frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \frac{30}{100}$.
$\frac{30}{100} + \frac{13}{100} = \frac{43}{100}$.

T: Write $\frac{43}{100}$ as a decimal.

S: 0.43.

T: Is this true? (Write $\frac{3}{10} + \frac{13}{100} = \frac{30}{100} + \frac{13}{100} = \frac{43}{100} = 0.43$.)

S: Yes.

Repeat the process with $\frac{2}{10} + \frac{36}{100}$ and $\frac{40}{100} + \frac{6}{10}$.

> **NOTES ON MULTIPLE MEANS OF REPRESENTATION:**
>
> After the initial use of multiplication to convert tenths to hundredths, many students may be able to do the conversion mentally. Encourage this shortcut because it is empowering. This is an important application of students' work with equivalence from Module 5, which leads to addition and subtraction of fractions with unlike denominators in Grade 5.
>
> If some students still struggle with the conversion, directly link the multiplication to the area model and place value chart.

Problem 3: Add tenths and hundredths with sums greater than 1. Express the sum as a decimal.

T: (Write $\frac{6}{10} + \frac{57}{100}$.) Read the expression.

S: 6 tenths + 57 hundredths.

T: Solve, and then explain your solution to your partner. (Two solution strategies are pictured below.)

MP.6

S: I changed 6 tenths to 60 hundredths and then made 1 by adding 50 hundredths, which I took out of each addend. That meant 10 hundredths and 7 hundredths were left to be added. The sum is $1\frac{17}{100}$.
→ I just added 60 hundredths and 57 hundredths to get 117 hundredths and then decomposed to get 100 hundredths and 17 hundredths. → I converted 6 tenths to 60 hundredths and then took out 40 hundredths from 57 hundredths to make 1 and added on the leftover 17 hundredths.

T: Write your answer as a decimal.

S: 1.17.

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Lesson 12:   Apply understanding of fraction equivalence to add tenths and hundredths.

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T: (Write $\frac{9}{10} + \frac{64}{100}$.)

T: Solve, and then share your solution strategy with a partner.

S: I used a number bond to decompose 64 hundredths into 10 hundredths and 54 hundredths to make 1. → I added to get 154 hundredths and decomposed the sum into 100 hundredths and 54 hundredths, or 1 and 54 hundredths.

T: Write your answer as a decimal.

S: 1.54.

Repeat the process with $\frac{2}{10} + \frac{91}{100}$ and $\frac{45}{100} + \frac{8}{10}$.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Apply understanding of fraction equivalence to add tenths and hundredths.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Lesson 12:   Apply understanding of fraction equivalence to add tenths and hundredths.

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Any combination of the questions below may be used to lead the discussion.

- How did the work in Problem 1 help to prepare you to solve Problem 2?

- In Problem 3(d), what do you notice about your answer? Can the answer be written using a unit other than hundredths? Does that apply to any solutions in Problem 4?

- In Problem 5, if the water and iodine are mixed together, can we just measure the amount of iodine in the large beaker? Explain.

- What have we learned before that made converting to like units so easy? What have we learned before that made adding tenths and hundredths so easy?

- How did the Application Problem connect to today's lesson?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Lesson 12: Apply understanding of fraction equivalence to add tenths and hundredths.

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Name _____   Date _____

1. Complete the number sentence by expressing each part using hundredths. Model using the place value chart, as shown in part (a).

ones	●	tenths	hundredths

a.   1 tenth + 5 hundredths = _____ hundredths

ones	●	tenths	hundredths

b.   2 tenths + 1 hundredth = _____ hundredths

ones	●	tenths	hundredths

c.   1 tenth + 12 hundredths = _____ hundredths

2. Solve by converting all addends to hundredths before solving.

a.   1 tenth + 3 hundredths = _____ hundredths + 3 hundredths = _____ hundredths

b.   5 tenths + 12 hundredths = _____ hundredths + _____ hundredths = _____ hundredths

c.   7 tenths + 27 hundredths = _____ hundredths + _____ hundredths = _____ hundredths

d.   37 hundredths + 7 tenths = _____ hundredths + _____ hundredths = _____ hundredths

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Lesson 12:   Apply understanding of fraction equivalence to add tenths and hundredths.

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3. Find the sum. Convert tenths to hundredths as needed. Write your answer as a decimal.

a. $\frac{2}{10} + \frac{8}{100}$

b. $\frac{13}{100} + \frac{4}{10}$

c. $\frac{6}{10} + \frac{39}{100}$

d. $\frac{70}{100} + \frac{3}{10}$

4. Solve. Write your answer as a decimal.

a. $\frac{9}{10} + \frac{42}{100}$

b. $\frac{70}{100} + \frac{5}{10}$

c. $\frac{68}{100} + \frac{8}{10}$

d. $\frac{7}{10} + \frac{87}{1v00}$

5. Beaker A has $\frac{63}{100}$ liter of iodine. It is filled the rest of the way with water up to 1 liter. Beaker B has $\frac{4}{10}$ liter of iodine. It is filled the rest of the way with water up to 1 liter. If both beakers are emptied into a large beaker, how much iodine does the large beaker contain?

Lesson 12:   Apply understanding of fraction equivalence to add tenths and hundredths.

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Name _____   Date _____

1. Complete the number sentence by expressing each part using hundredths. Use the place value chart to model.

ones	•	tenths	hundredths

1 tenth + 9 hundredths = _____ hundredths

2. Find the sum. Write your answer as a decimal.

$$\frac{4}{10} + \frac{73}{100}$$

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Lesson 12:   Apply understanding of fraction equivalence to add tenths and hundredths.

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Name _____   Date _____

1. Complete the number sentence by expressing each part using hundredths. Model using the place value chart, as shown in part (a).

ones	•	tenths	hundredths

a. 1 tenth + 8 hundredths = _____ hundredths

ones	•	tenths	hundredths

b. 2 tenths + 3 hundredths = _____ hundredths

ones	•	tenths	hundredths

c. 1 tenth + 14 hundredths = _____ hundredths

2. Solve by converting all addends to hundredths before solving.

a. 1 tenth + 2 hundredths = _____ hundredths + 2 hundredths = _____ hundredths

b. 4 tenths + 11 hundredths = _____ hundredths + _____ hundredths = _____ hundredths

c. 8 tenths + 25 hundredths = _____ hundredths + _____ hundredths = _____ hundredths

d. 43 hundredths + 6 tenths = _____ hundredths + _____ hundredths = _____ hundredths

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Lesson 12: Apply understanding of fraction equivalence to add tenths and hundredths.

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3. Find the sum. Convert tenths to hundredths as needed. Write your answer as a decimal.

a. $\frac{3}{10} + \frac{7}{100}$

b. $\frac{16}{100} + \frac{5}{10}$

c. $\frac{5}{10} + \frac{40}{100}$

d. $\frac{20}{100} + \frac{8}{10}$

4. Solve. Write your answer as a decimal.

a. $\frac{5}{10} + \frac{53}{100}$

b. $\frac{27}{100} + \frac{8}{10}$

c. $\frac{4}{10} + \frac{78}{100}$

d. $\frac{98}{100} + \frac{7}{10}$

5. Cameron measured $\frac{65}{100}$ inch of rainwater on the first day of April. On the second day of April, he measured $\frac{83}{100}$ inch of rainwater. How many total inches of rainwater did Cameron measure on the first two days of April?

Lesson 12: Apply understanding of fraction equivalence to add tenths and hundredths.

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ones	●	tenths	hundredths

area model and place value chart

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Lesson 12:   Apply understanding of fraction equivalence to add tenths and hundredths.

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Lesson 13

Objective: Add decimal numbers by converting to fraction form.

Suggested Lesson Structure

■ Fluency Practice   (8 minutes)
■ Concept Development   (42 minutes)
■ Student Debrief   (10 minutes)

Total Time   **(60 minutes)**
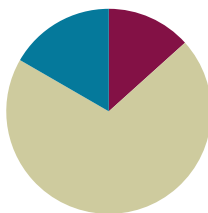
Fluency Practice (8 minutes)

▪ Order Decimal Numbers **4.NF.7**   (4 minutes)
▪ Write in Decimal and Fraction Notation **4.NF.5**   (4 minutes)

Order Decimal Numbers (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 11.

T: (Write 0.44, $\frac{1}{10}$, and 0.5.) Arrange the numbers in order from least to greatest.

S: (Write $\frac{1}{10}$, 0.44, and 0.5.)

Continue with the following possible sequence:

▪   0.6, 0.55, $\frac{16}{10}$, $\frac{65}{100}$, 0.87, 0.1, 0.77, 0.88

▪ $\frac{87}{10}$, 6 ones and 8 hundredths, $\frac{68}{100}$, 8 and 6 tenths, $\frac{680}{100}$, 6.86

Write in Decimal and Fraction Notation (4 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 7.

T: (Write 25.34.) Say the number.

S: 25 and 34 hundredths.

T: Write 25 and 34 hundredths in decimal expanded form without multiplication.

S: (Write 20 + 5 + 0.3 + 0.04.)

> **NOTES ON MULTIPLE MEANS OF ACTION AND EXPRESSION:**
>
> Adjust the Order Decimal Numbers fluency activity so that English language learners gain more practice in oral comprehension and transcribing. Rather than writing the numbers on the board, give directions, such as, "Arrange the following numbers in order from least to greatest: 44 hundredths, 1 tenth, 5 tenths." If desired, give an additional direction, such as, "Write some numbers as decimals and some as fractions." Students who are challenged by writing at such a fast pace may enjoy ordering cards (with decimals and fractions) as used in Lesson 11.

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Lesson 13:   Add decimal numbers by converting to fraction form.

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T: Write 25 and 34 hundredths in decimal expanded form with multiplication.

S: (Write 25.34 = (2 × 10) + (5 × 1) + (3 × 0.1) + (4 × 0.01).)

T: Write 25 and 34 hundredths in fraction expanded form with multiplication.

S: (Write 25.34 = (2 × 10) + (5 × 1) + (3 × $\frac{1}{10}$) + (4 × $\frac{1}{100}$).)

Continue the process for the following possible sequence: 28.07 and 452.70.

Concept Development (42 minutes)

Materials: (S) Personal white board

Problem 1: Add two decimal numbers less than 1 by converting to fraction form.

T: (Write 0.3 + 0.57.) Say the expression.

S: 3 tenths plus 57 hundredths.

T: Let's use what we know to add. Convert 3 tenths + 57 hundredths to fraction form.

S: (Write $\frac{3}{10} + \frac{57}{100}$.)

T: Solve.

S: $\frac{3}{10} = \frac{30}{100}$. So, $\frac{30}{100} + \frac{57}{100} = \frac{87}{100}$.

T: Write your answer as a decimal.

S: 0.87.

T: Write 0.5 + 0.64. Convert to fraction form.

S: (Write $\frac{5}{10} + \frac{64}{100}$.)

T: Solve.

S: $\frac{5}{10} = \frac{50}{100}$. So, $\frac{50}{100} + \frac{64}{100} = \frac{114}{100}$. That's more than 1.

T: Convert to a mixed number.

S: $\frac{114}{100} = 1\frac{14}{100}$. → I solved by decomposing an addend to make 1 using a number bond. The answer is $1\frac{14}{100}$.

T: Write your answer as a decimal.

S: 1.14.

T: Add 0.30 to 0.5.

S: $0.30 = \frac{30}{100}$. $0.5 = \frac{5}{10} = \frac{50}{100}$. So, $\frac{30}{100} + \frac{50}{100} = \frac{80}{100}$. $\frac{80}{100}$ is the same as $\frac{8}{10}$, so the answer is 0.80 or 0.8. → I converted hundredths to tenths instead before adding! $\frac{30}{100} = \frac{30 \div 10}{100 \div 10} = \frac{3}{10}$. So, $\frac{3}{10} + \frac{5}{10} = \frac{8}{10} = 0.8$.

Repeat the process with the following possible sequence: 0.2 + 0.31 and 0.29 + 0.8.

MP.2

NOTES ON CONVERTING DECIMALS TO FRACTIONS TO ADD:

While converting decimals to fractions before adding may not seem as quick as just adding decimals (e.g., 0.3 and 0.57), doing so strengthens student understanding of the fraction and decimal relationship, increases their ability to think flexibly, and prepares them for greater success with fractions and decimals in Grade 5.

$0.3 + 0.57 = \frac{3}{10} + \frac{57}{100}$

$= \frac{30}{100} + \frac{57}{100}$

$= \frac{87}{100}$

$= 0.87$

$0.5 + 0.64 = \frac{5}{10} + \frac{64}{100}$

$= \frac{50}{100} + \frac{64}{100}$

$\frac{50}{100} \quad \frac{14}{100}$

$= 1\frac{14}{100} = 1.14$

Lesson 13: Add decimal numbers by converting to fraction form.

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Problem 2: Add two decimal numbers involving whole numbers and like fractional units by converting to fractional form.

T: (Write 6.8 + 5.7.) Rewrite this expression as the sum of two mixed numbers.

S: (Write $6\frac{8}{10} + 5\frac{7}{10}$.)

T: What do you know about mixed number addition to help you solve this problem?

S: I can add the whole numbers and then add the tenths. → I can add ones to ones and then add the fractions because they have the same denominator.

T: Solve with your partner.

S: $6\frac{8}{10} + 5\frac{7}{10} = (6+5) + \left(\frac{8}{10} + \frac{7}{10}\right) = 11\frac{15}{10} = 12\frac{5}{10}$.
(Another possible solution is shown to the right.)

T: Rewrite your number sentence in decimal form.

S: 6.8 + 5.7 = 12.5.

T: (Write 4.28 + 2.97.) Rewrite this expression as the sum of two mixed numbers.

S: (Write $4\frac{28}{100} + 2\frac{97}{100}$.)

T: Solve with your partner. (One possible solution is shown to the right.)

T: Rewrite your number sentence in decimal form.

S: 4.28 + 2.97 = 7.25.

$6.8 + 5.7 = 6\frac{8}{10} + 5\frac{7}{10}$

$= 11\frac{15}{10}$

$1 \quad \frac{5}{10}$

$= 12\frac{5}{10}$

$6.8 + 5.7 = 12.5$

$4.28 + 2.97 = 4\frac{28}{100} + 2\frac{97}{100}$

$= 6\frac{125}{100} = 7\frac{25}{100}$

$1 \quad \frac{25}{100}$

$4.28 + 2.97 = 7.25$

Problem 3: Add two decimal numbers involving whole numbers, tenths, and hundredths with unlike units by converting to fractional form.

T: (Write 3.5 + 2.49.) Convert this expression to fraction form as the sum of two mixed numbers.

S: (Write $3\frac{5}{10} + 2\frac{49}{100}$.)

T: What do you know about mixed number addition to help you solve this problem?

S: I can add ones to ones and then add the fractions after I change them to like units. → I have to change the tenths to hundredths to add the fractions.

T: Solve with your partner.

S: $3\frac{50}{100} + 2\frac{49}{100} = 5\frac{50}{100} + \frac{49}{100} = 5\frac{99}{100}$.

T: Rewrite your number sentence in decimal form.

S: 3.5 + 2.49 = 5.99.

$3.5 + 2.49 = 3\frac{50}{100} + 2\frac{49}{100}$

$= 5\frac{50}{100} + \frac{49}{100}$

$= 5\frac{99}{100}$

$3.5 + 2.49 = 5.99$

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Lesson 13: Add decimal numbers by converting to fraction form.

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T: (Write 5.6 + 4.53.) Rewrite this expression as the sum of mixed numbers in fraction form.

S: (Write $5\frac{6}{10} + 4\frac{53}{100}$.)

T: Work with a partner to solve. After you have found the sum in fraction form, rewrite the decimal number sentence.

T: (Allow students time to work, and then present two or three alternate solutions as exemplified below.) Analyze and discuss the following solution strategies with your partner.

S: The first solution shows adding like units and decomposing the sum of the hundredths into 1 and 13 hundredths. → The second solution shows decomposing $\frac{53}{100}$ to take out $\frac{4}{10}$ to make 1. They then added 9 ones with the 1 they made from 6 tenths and 4 tenths to get 10 ones and 13 hundredths. → The third solution shows converting tenths to hundredths in one step. Then, they decomposed the hundredths to make 1 from 60 hundredths and 40 hundredths. 6 ones and 4 ones is 10 ones with 13 hundredths. → All of them show the same decimal number sentence.

T: Yes, remember that there are multiple solution strategies that we learned when adding fractions that we can use here when adding decimal fractions.

Repeat with 3.82 + 19.6.

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

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Lesson 13:   Add decimal numbers by converting to fraction form.

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Student Debrief (10 minutes)

Lesson Objective: Add decimal numbers by converting to fraction form.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Explain to your partner the process of adding two mixed numbers. Why do we need to convert to like units?
- What other conversion could you have used for Problems 2(a) and 2(c)?
- For Problems 2(b) and 2(d), explain how in the solution you could make 1 before adding the hundredths together.
- What was the added complexity of Problem 2 in the Problem Set? How did your prior knowledge of adding mixed numbers from Module 5 help to make this task easier?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Lesson 13:   Add decimal numbers by converting to fraction form.

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Name _____   Date _____

1. Solve. Convert tenths to hundredths before finding the sum. Rewrite the complete number sentence in decimal form. Problems 1(a) and 1(b) are partially completed for you.

a. $2\frac{1}{10} + \frac{3}{100} = 2\frac{10}{100} + \frac{3}{100} = $ _____ $2.1 + 0.03 = $ _____	b. $2\frac{1}{10} + 5\frac{3}{100} = 2\frac{10}{100} + 5\frac{3}{100} = $ _____
c. $3\frac{24}{100} + \frac{7}{10}$	d. $3\frac{24}{100} + 8\frac{7}{10}$

2. Solve. Then, rewrite the complete number sentence in decimal form.

a. $6\frac{9}{10} + 1\frac{10}{100}$	b. $9\frac{9}{10} + 2\frac{45}{100}$
c. $2\frac{4}{10} + 8\frac{90}{100}$	d. $6\frac{37}{100} + 7\frac{7}{10}$

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3. Solve by rewriting the number sentence in fraction form. After solving, rewrite the complete number sentence in decimal form.

a. 6.4 + 5.3	b. 6.62 + 2.98
c. 2.1 + 0.94	d. 2.1 + 5.94
e. 5.7 + 4.92	f. 5.68 + 4.9
g. 4.8 + 3.27	h. 17.6 + 3.59

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Lesson 13:   Add decimal numbers by converting to fraction form.

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Name _____   Date _____

Solve by rewriting the number sentence in fraction form. After solving, rewrite the complete number sentence in decimal form.

1.   7.3 + 0.95

2.   8.29 + 5.9

Name _____   Date _____

1. Solve. Convert tenths to hundredths before finding the sum. Rewrite the complete number sentence in decimal form. Problems 1(a) and 1(b) are partially completed for you.

a. $5\frac{2}{10} + \frac{7}{100} = 5\frac{20}{100} + \frac{7}{100} =$ _____ $5.2 + 0.07 =$ _____	b. $5\frac{2}{10} + 3\frac{7}{100} = 8\frac{20}{100} + \frac{7}{100} =$ _____
c. $6\frac{5}{10} + \frac{1}{100}$	d. $6\frac{5}{10} + 7\frac{1}{100}$

2. Solve. Then, rewrite the complete number sentence in decimal form.

a. $4\frac{9}{10} + 5\frac{10}{100}$	b. $8\frac{7}{10} + 2\frac{65}{100}$
c. $7\frac{3}{10} + 6\frac{87}{100}$	d. $5\frac{48}{100} + 7\frac{8}{10}$

Lesson 13:   Add decimal numbers by converting to fraction form.

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3. Solve by rewriting the number sentence in fraction form. After solving, rewrite the complete number sentence in decimal form.

a. $2.1 + 0.87 = 2\frac{1}{10} + \frac{87}{100}$	b. $7.2 + 2.67$
c. $7.3 + 1.8$	d. $7.3 + 1.86$
e. $6.07 + 3.93$	f. $6.87 + 3.9$
g. $8.6 + 4.67$	h. $18.62 + 14.7$

Lesson 13:   Add decimal numbers by converting to fraction form.

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Lesson 14

Objective: Solve word problems involving the addition of measurements in decimal form.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
□ Concept Development   (38 minutes)
■ Student Debrief   (10 minutes)
  Total Time   **(60 minutes)**
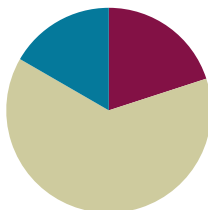
Fluency Practice (12 minutes)

▪ State the Value of the Coins **4.MD.2**   (2 minutes)
▪ Add Decimals **4.NF.5**   (5 minutes)
▪ Write in Decimal and Fraction Notation **4.NF.5**   (5 minutes)

State the Value of the Coins (2 minutes)

Materials: (S) Personal white board

Note: This fluency activity prepares students for Lessons 15–16.

    T: (Write 1 dime = __ ¢.) What is the value of 1 dime?
    S: 10¢.
    T: 2 dimes?
    S: 20¢.
    T: 3 dimes?
    S: 30¢.
    T: 8 dimes?
    S: 80¢.
    T: (Write 10 dimes = __ dollar.) Write the number sentence.
    S: (Write 10 dimes = 1 dollar.)
    T: (Write 20 dimes = __ dollars.) Write the number sentence.
    S: (Write 20 dimes = 2 dollars.)
    T: (Write 1 penny = __ ¢.) What is the value of 1 penny?
    S: 1¢.

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T: 2 pennies?

S: 2¢.

T: 3 pennies?

S: 3¢.

T: 9 pennies?

S: 9¢.

T: (Write 7 pennies = __¢.) Write the number sentence.

S: (Write 7 pennies = 7¢.)

Add Decimals (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 13.

> **NOTES ON MULTIPLE MEANS OF ENGAGEMENT:**
>
> English language learners and others may benefit from a reminder, such as a poster, personal dictionary, or word wall, that defines and provides examples of *standard form, fraction form, unit form,* and *decimal form.* Examples may provide clarity for the Add Decimals fluency activity.

T: (Write 4 tens + 2 ones.) Say the addition sentence in standard form.

S: 40 + 2 = 42.

T: (Write $\frac{4}{10} + \frac{2}{100} = \underline{}\frac{}{100}$.) Write the number sentence.

S: (Write $\frac{4}{10} + \frac{2}{100} = \frac{42}{100}$.)

T: (Write $\frac{4}{10} + \frac{2}{100} = \frac{42}{100}$.) Write the number sentence in decimal form.

S: (Write 0.4 + 0.02 = 0.42.)

Continue with the following possible sequence: $\frac{8}{10} + \frac{3}{100}$, $\frac{13}{100} + \frac{2}{10}$, $\frac{5}{10} + \frac{30}{100}$, $\frac{40}{100} + \frac{4}{10}$, $\frac{7}{10} + \frac{30}{100}$, and $\frac{8}{10} + \frac{37}{100}$.

Write in Decimal and Fraction Notation (5 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 12.

T: (Write 36.79.) Say the number.

S: 36 and 79 hundredths.

T: Write 36 and 79 hundredths in decimal expanded form without multiplication.

S: (Write 36.79 = 30 + 6 + 0.7 + 0.09.)

T: (Write 36.79 = (_ × 10) + (_ × 1) + (_ × 0.1) + (_ × 0.01).) Complete the number sentence.

S: (Write 36.79 = (3 × 10) + (6 × 1) + (7 × 0.1) + (9 × 0.01).)

T: Write 36 and 79 hundredths in fraction expanded form with multiplication.

S: (Write $36\frac{79}{100} = (3 \times 10) + (6 \times 1) + (7 \times \frac{1}{10}) + (9 \times \frac{1}{100})$.)

Continue with the following possible sequence: 34.09 and 734.80.

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Lesson 14:   Solve word problems involving the addition of measurements in decimal form.

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Concept Development (38 minutes)

Materials: (S) Personal white board, Problem Set

Suggested Delivery of Instruction for Solving This Lesson's Word Problems

1. Model the problem.

Have two pairs of students model the problem at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give students two minutes to finish their work on that question, sharing their work and thinking with a peer. All should then write their equations and statements of the answer.

3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solutions.

Problem 1

Barrel A contains 2.7 liters of water. Barrel B contains 3.09 liters of water. Together, how much water do the two barrels contain?
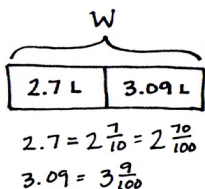
> **NOTES ON**
> **MULTIPLE MEANS**
> **OF REPRESENTATION:**
>
> In today's lesson, students apply their skill with adding decimals by first converting them to fraction form. The first two problems are single-step problems. Encourage students to use the RDW process because, in doing so, they again realize that part–whole relationships are the same whether the parts are whole numbers, fractions, or mixed numbers.

The first problem of the day starts at a simple level to give students the opportunity to simply apply their skill with converting decimal numbers to fraction form to solve a word problem. Students solve this problem by converting 2.7 liters and 3.09 liters to fractional form, converting tenths to hundredths, and adding the mixed numbers. Remind students to convert their answers to decimal form when writing their statements.

Lesson 14: Solve word problems involving the addition of measurements in decimal form.

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Problem 2

Alissa ran a distance of 15.8 kilometers one week and 17.34 kilometers the following week. How far did she run in the two weeks?

Problem 2 invites various solution strategies because the sum of the fractions is greater than 1, and the whole numbers are larger. In Solution A, students add like units and decompose by drawing a number bond to show $\frac{114}{100}$ as $1 + \frac{14}{100}$ and then adding 32. In Solutions B and C, students use different methods of breaking apart $\frac{34}{100}$ to add up to make 1.

Problem 3

An apple orchard sold 140.5 kilograms of apples in the morning and 15.85 kilograms more apples in the afternoon than in the morning. How many total kilograms of apples were sold that day?

Tape diagram:
M: 140.5 kg
A: 140.5 kg | 15.85 kg
} ?
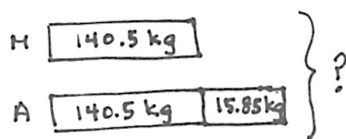
Solution A

$$140\frac{5}{10} + 15\frac{85}{100} = 155\frac{50}{100} + \frac{85}{100}$$

$$= 155\frac{135}{100}$$

$$= 156\frac{35}{100}$$

$$140\frac{5}{10} + 156\frac{35}{100} = 296\frac{50}{100} + \frac{35}{100}$$

$$= 296\frac{85}{100}$$

Solution B

$$\left(2 \times 140\frac{5}{10}\right) + 15.85 \quad = 280\frac{10}{10} + 15\frac{85}{100}$$

$$= 296\frac{85}{100}$$

The apple orchard sold 296.85 kilograms of apples.

This problem brings the additional complexity of two steps. Students solve this problem by converting 140.5 kilograms and 15.85 kilograms to fractional form, converting tenths to hundredths, and then adding the mixed numbers. Remind students to convert their answers to decimal form and to include the labeled units in their answers. Solution A shows solving for the number of kilograms sold in the afternoon and then solving for the total number of kilograms sold in the day by adding the kilograms of apples from the morning with those from the afternoon. In Solution B, the number of kilograms sold in the morning is multiplied by 2, and then the additional kilograms sold in the afternoon are added.

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Problem 4

A team of three ran a relay race. The final runner's time was the fastest, measuring 29.2 seconds. The middle runner's time was 1.89 seconds slower than the final runner's. The starting runner's time was 0.9 seconds slower than the middle runner's. What was the team's total time for the race?

This problem involves two additional challenges. First, students must realize that when a runner goes slower, there is more time added on. Second, to find the starting runner's time, students must add the 9 tenths second to the middle runner's time. Notice the difference in Solution A's and Solution B's models. In Solution A, the student finds the time of each individual runner, first adding 1.89 seconds to 29.2 seconds and then adding 0.9 seconds to that sum to find the time of the starting runner. On the other hand, Solution B shows how a student solves by thinking of the starting runner in relationship to the final runner. As a result, she is able to discern the 3 units of 29.2 seconds, multiplies 29.2 by 3, adds $1\frac{89}{100} + 1\frac{89}{100} + \frac{9}{10}$, and adds the two sums together.

Student Debrief (10 minutes)

Lesson Objective: Solve word problems involving the addition of measurements in decimal form.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- ▪ What was the added complexity of Problem 3? What about Problem 4?
- ▪ Explain the strategies that you used to solve Problems 3 and 4.

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

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Lesson 14: Solve word problems involving the addition of measurements in decimal form.

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Name _____   Date _____

1. Barrel A contains 2.7 liters of water. Barrel B contains 3.09 liters of water. Together, how much water do the two barrels contain?

2. Alissa ran a distance of 15.8 kilometers one week and 17.34 kilometers the following week. How far did she run in the two weeks?

Lesson 14:   Solve word problems involving the addition of measurements in decimal form.

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3. An apple orchard sold 140.5 kilograms of apples in the morning and 15.85 kilograms more apples in the afternoon than in the morning. How many total kilograms of apples were sold that day?
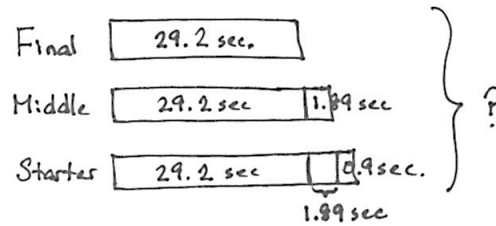
4. A team of three ran a relay race. The final runner's time was the fastest, measuring 29.2 seconds. The middle runner's time was 1.89 seconds slower than the final runner's. The starting runner's time was 0.9 seconds slower than the middle runner's. What was the team's total time for the race?

Name _____   Date _____

Elise ran 6.43 kilometers on Saturday and 5.6 kilometers on Sunday. How many total kilometers did she run on Saturday and Sunday?

Name _____   Date _____

1. The snowfall in Year 1 was 2.03 meters. The snowfall in Year 2 was 1.6 meters. How many total meters of snow fell in Years 1 and 2?

2. A deli sliced 22.6 kilograms of roast beef one week and 13.54 kilograms the next. How many total kilograms of roast beef did the deli slice in the two weeks?

Lesson 14: Solve word problems involving the addition of measurements in decimal form.

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3. The school cafeteria served 125.6 liters of milk on Monday and 5.34 more liters of milk on Tuesday than on Monday. How many total liters of milk were served on Monday and Tuesday?

4. Max, Maria, and Armen were a team in a relay race. Max ran his part in 17.3 seconds. Maria was 0.7 seconds slower than Max. Armen was 1.5 seconds slower than Maria. What was the total time for the team?

Lesson 14: Solve word problems involving the addition of measurements in decimal form.

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Topic E
Money Amounts as Decimal Numbers

4.MD.2, 4.NF.5, 4.NF.6

Focus Standard:	4.MD.2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
Instructional Days:	2	
Coherence -Links from:	G2–M7	Problem Solving with Length, Money, and Data
	G3–M5	Fractions as Numbers on the Number Line
-Links to:	G5–M2	Multi-Digit Whole Number and Decimal Fraction Operations

In Topic E, students work with money amounts as decimal numbers, applying what they have come to understand about decimals.

Students recognize 1 penny as $\frac{1}{100}$ dollar, 1 dime as $\frac{1}{10}$ dollar, and 1 quarter as $\frac{25}{100}$ dollar in Lesson 15. They apply their understanding of tenths and hundredths to express money amounts in both fraction and decimal forms. Students use this understanding to decompose varying configurations and forms of dollars, quarters, dimes, and pennies and express each as a decimal fraction and decimal number. They then expand this skill to include money amounts greater than a dollar in decimal form.
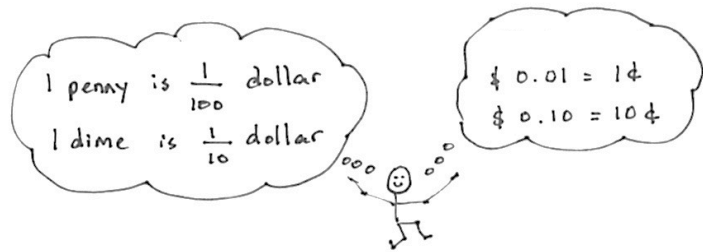
In Lesson 16, students continue their work with money and apply their understanding that only like units can be added. They solve word problems involving money using all four operations (**4.MD.2**). Addition and subtraction word problems are computed using dollars and cents in unit form. Multiplication and division word problems are computed using cents in unit form. All answers are converted from unit form into decimal form, using the dollar symbol as the unit.

Jack has 2 quarters and 7 dimes. Jim has 1 dollar, 3 quarters, and 6 pennies. How much money do they have together? Write your answer as a decimal.

Jack | 50¢ | 70¢ |

They have $3.01 together.

Jim | $1 | 75¢ | 6¢ |

Solution A

1 dollar 20 cents + 1 dollar 81 cents

= 2 dollars 101 cents
　　　　　　　　 ∧
　　　　　　 100 1

= 3 dollars 1 cent

= $3.01

Solution B

1 dollar 20 cents + 1 dollar 81 cents
　　　　　　　　　　　　　　 ∧
　　　　　　　　　　　　 80 1

= 3 dollars 1 cent

= $3.01

A Teaching Sequence Toward Mastery of Money Amounts as Decimal Numbers

Objective 1: Express money amounts given in various forms as decimal numbers.
　　　　　　　　(Lesson 15)

Objective 2: Solve word problems involving money.
　　　　　　　　(Lesson 16)

Lesson 15

Objective: Express money amounts given in various forms as decimal numbers.

Suggested Lesson Structure

■ Fluency Practice   (10 minutes)
■ Application Problem   (4 minutes)
■ Concept Development   (36 minutes)
■ Student Debrief   (10 minutes)

Total Time   **(60 minutes)**
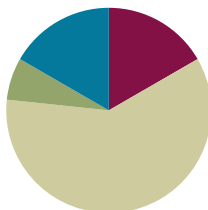
Fluency Practice (10 minutes)

- Add Fractions **4.NF.5**   (5 minutes)
- State the Value of the Coins **4.MD.2**   (5 minutes)

Add Fractions (5 minutes)

Note: This fluency activity reviews Lesson 13.

T: (Write 90 + 7 = ___.) Say the addition sentence in unit form.

S: 9 tens + 7 ones = 97 ones.

T: (Write $\frac{9}{10} + \frac{7}{100} = \frac{}{100}$.) Say the addition sentence in unit form.

S: 9 tenths + 7 hundredths = 97 hundredths.

Continue with the following possible sequence: 40 + 8 and $\frac{4}{10} + \frac{8}{100}$; 20 + 9 and $\frac{2}{10} + \frac{9}{100}$.

T: (Write 70 + 18 = ___.) Say the addition sentence in unit form.

S: 7 tens + 18 ones = 88 ones.

T: (Write $\frac{7}{10} + \frac{18}{100} = \frac{}{100}$.) Say the addition sentence in unit form.

S: 7 tenths + 18 hundredths = 88 hundredths.

Continue with the following possible sequence: 60 + 13 and $\frac{6}{10} + \frac{13}{100}$; 30 + 29 and $\frac{3}{10} + \frac{29}{100}$.

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State the Value of the Coins (5 minutes)

Note: This fluency activity prepares students for Lessons 15–16.

T: (Write 10¢ = 1 _____.) What coin has a value of 10 cents?
S: 1 dime.
T: 90¢ is the same as how many dimes?
S: 9 dimes.
T: (Write 25¢ = 1 _____.) What coin has a value of 25 cents?
S: 1 quarter.
T: 50¢ is the same as how many quarters?
S: 2 quarters.
T: 75¢ is the same as how many quarters?
S: 3 quarters.
T: 100¢ is the same as how many quarters?
S: 4 quarters.
T: What is the value of 2 quarters?
S: 50 cents.
T: What is the total value of 2 quarters and 2 dimes?
S: 70 cents.
T: What is the total value of 2 quarters and 6 dimes?
S: 110 cents.

Continue with the following possible sequence: 1 quarter 5 dimes, 3 quarters 2 dimes, 2 quarters 7 dimes, and 3 quarters 2 dimes 1 penny.

Application Problem (4 minutes)

At the end of the day, Cameron counted the money in his pockets. He counted 7 pennies, 2 dimes, and 2 quarters. Tell the amount of money, in cents, that was in Cameron's pockets.

Note: This Application Problem builds on the previous knowledge of money from Grade 2 Module 7, where students solved word problems involving money. In the last two lessons of this module, students extend their prior work with money amounts to think of the number of dollars and cents units and record money amounts using decimals.

Lesson 15: Express money amounts given in various forms as decimal numbers.

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Concept Development (36 minutes)

Materials: (S) Personal white board

Problem 1: Express pennies, dimes, and quarters as fractional parts of a dollar.

T: How many pennies are in 1 dollar?

S: 100 pennies.

T: $\frac{1}{100}$ dollar is equal to how many cents?

S: 1 cent.

T: (Write 1¢ = $\frac{1}{100}$ dollar.)

T: We can write 1 hundredth dollar using a decimal. Write $\frac{1}{100}$ in decimal form.

S: (Write 0.01.)

T: Place the dollar sign before the ones. (Write 1¢ = $\frac{1}{100}$ dollar = $0.01.) (Point to the number sentence.) We can read $0.01 as 1 cent.

MP.2

T: (Show 7 pennies.) 7 pennies are how many cents?

S: 7 cents.

T: What fraction of a dollar is 7 cents?

S: $\frac{7}{100}$ dollar.

T: Write a number sentence to show the value of 7 pennies as cents, as a fraction of a dollar, and in decimal form.

S: (Write 7¢ = $\frac{7}{100}$ dollar = $0.07.)

Repeat writing equivalent number sentences for 31, 80, and 100 pennies.

T: A dime also represents a fractional part of a dollar. How many dimes are in a dollar?

S: 10 dimes.

T: Draw a tape diagram to show how many dimes are needed to make 1 dollar.

S: (Draw.)

T: What fraction of a dollar is 1 dime?

S: $\frac{1}{10}$ dollar.

T: $\frac{1}{10}$ dollar is equal to how many cents?

S: 10 cents.

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Lesson 15:   Express money amounts given in various forms as decimal numbers.

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T: (Write 10¢ = $\frac{1}{10}$ dollar.) Write $\frac{1}{10}$ dollar as an equivalent decimal using the dollar sign to tell the unit.

S: (Write 10¢ = $\frac{1}{10}$ dollar = \$0.10.)

Repeat writing equivalent number sentences for 8 dimes and 10 dimes.

T: With your partner, draw a tape diagram to show how many quarters equal 1 dollar. Write a number sentence to show the equivalence of the value of 1 quarter written as cents, as a fraction of a dollar, and as a decimal.

Expect many students to write $\frac{1}{4}$ dollar, which is correct.

To write the value of 1 quarter as a decimal, remind students to write an equivalent fraction using 100 as the denominator so that students show 25¢ = $\frac{25}{100}$ dollar = \$0.25.

Problem 2: Express the total value of combinations of pennies, dimes, and quarters in fraction and decimal form.

T: (Write 7 dimes 2 pennies.) What is the value of 7 dimes 2 pennies expressed in cents?

S: 72 cents.

T: What number sentence did you use to find that value?

S: 70 + 2 = 72. ➔ (7 × 10¢) + 2¢ = 72¢.

T: What fraction of a dollar is 72 cents?

S: $\frac{72}{100}$ dollar.

T: On your personal white board, express $\frac{72}{100}$ dollar in decimal form, using the dollar sign.

S: \$0.72.

Repeat with 2 quarters 3 dimes 6 pennies.

T: (Write 3 quarters 4 dimes.) What is the value of 3 quarters 4 dimes expressed in cents? (Allow students time to work.)

S: 115 cents.

T: How did you find that value?

S: I counted by 25 three times and then counted up by 10 four times. ➔ (3 × 25) + (4 × 10) = 115. ➔ 75¢ + 40¢ = 115¢.

T: Do we have more or less than a dollar?

S: More.

T: What fraction of a dollar is 115¢?

S: $\frac{115}{100}$ dollars. ➔ $1\frac{15}{100}$ dollars.

> **NOTE ON READING DECIMALS AND FRACTIONS OF A UNIT:**
>
> How do people read fractions and decimals? Make sure to offer English language learners and others valuable practice reading fractions and decimals correctly. To make close connections between decimals, fractions and units, read numbers such as 0.33 meter as "thirty-three hundredths meter" and $\frac{1}{100}$ dollar as "one hundredth dollar." Also, share common ways in which decimals and fractions are read daily, such as "zero point thirty-three meters" or "one hundredth of a dollar." Model for and guide students toward the preferred method of reading decimals and fractions based on the situation.

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Lesson 15:   Express money amounts given in various forms as decimal numbers.

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T: Express $1\frac{15}{100}$ dollars as a decimal, using the dollar sign to express the unit.

S: $1.15.

Repeat the process with 5 quarters 7 pennies.

T: What did we do when finding the value of a set of coins?

S: We multiplied by 25 to find the value of the quarters and by 10 to find the value of the dimes.
→ We just used multiplication and addition with whole numbers, and then we expressed our answer as a fraction of a dollar and in decimal form with the dollar sign.

Problem 3: Find the sum of two sets of bills and cents using whole number calculations and unit form.

T: (Write 6 dollars 1 dime 7 pennies + 8 dollars 1 quarter.) Let's rewrite each addend as dollars and cents.

S: 6 dollars 17 cents + 8 dollars 25 cents.

T: Let's add like units to find the sum. 6 dollars + 8 dollars is …?

S: 14 dollars.

T: 17 cents + 25 cents is …?

S: 42 cents.

T: Write the complete number sentence on your board.

S: 6 dollars 17 cents + 8 dollars 25 cents = 14 dollars 42 cents.

T: Write your sum in decimal form using the dollar sign to designate the unit.

S: $14.42.

T: (Write 5 dollars 3 dimes 17 pennies + 4 dollars 3 quarters 2 dimes.) Work with a partner to write an expression showing each addend in unit form as dollars and cents.

S: 5 dollars 47 cents + 4 dollars 95 cents.

T: Add dollars with dollars and cents with cents to find the sum.

S: 9 dollars 142 cents. → 10 dollars 42 cents.

T: Why do these two different solutions show the same answer? Talk to your partner.

S: 142 cents is the same as 1 dollar 42 cents. We changed 9 dollars to 10 dollars (Solution A). → We completed the dollar. 95 cents + 47 cents is the same as 95 + 5 + 42 or 100 + 42, which is 1 dollar and 42 cents (Solution B). → We added to get 142 cents and then decomposed the cents into 1 dollar and 42 cents (Solution A).

$(3 \times 10) + 17 = 47$
$(3 \times 25) + 20 = 95$
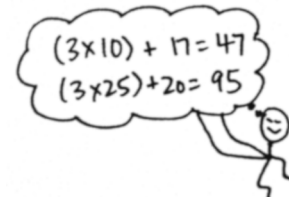
5 dollars 3 dimes 17 pennies + 4 dollars 3 quarters 2 dimes

Solution A
5 dollars 47 cents + 4 dollars 95 cents
= 9 dollars 142 cents
      100 42
= 10 dollars 42 cents
= $ 10.42

Solution B
5 dollars 47 cents + 4 dollars 95 cents
= 9 dollars 47 cents + 95 cents
      42 5
= 10 dollars 42 cents
= $ 10.42

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Give students additional practice as necessary. This final component is directly applied in Lesson 16 to word problems.

- 10 dollars 1 quarter 3 dimes + 3 dollars 5 dimes 14 pennies
- 15 dollars 7 dimes 6 pennies + 2 quarters 23 pennies

Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students should solve these problems using the RDW approach used for Application Problems.

Student Debrief (10 minutes)

Lesson Objective: Express money amounts given in various forms as decimal numbers.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.
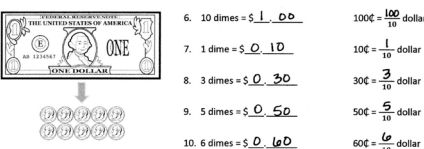
- How is money related to decimals and fractions? How is it different? Think about why we would write money in expanded form.
- I have $\frac{2}{5}$ dollar in my pocket. Use what you know about equivalent fractions to determine how many cents I have. What are some possible combinations of coins that may be in my pocket? Do not forget about nickels!

- Are $1 and $1.00 equal? Are $0.1 and $0.10 equal? Are all these forms correct? Which form may not be used frequently and why?

- How did the Application Problem prepare you for today's lesson?

- How might dimes be expressed as fractions differently than as tenths of a dollar? Use an example from Problems 6–10.

- How can the fraction of a dollar for Problem 13 be simplified?

- When adding fractions and whole numbers, we sometimes complete the next whole or the next hundred to simplify the addition. How, in Problem 20, could you decompose 8 dimes to simplify the addition?

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

Lesson 15:   Express money amounts given in various forms as decimal numbers.

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Name _____   Date _____

1. 100 pennies = $___._____   100¢ = ——— dollar
                                              100

2. 1 penny = $___._____   1¢ = ——— dollar
                                            100

3. 6 pennies = $___._____   6¢ = ——— dollar
                                            100

4. 10 pennies = $___._____   10¢ = ——— dollar
                                            100

5. 26 pennies = $___._____   26¢ = ——— dollar
                                            100

6. 10 dimes = $___._____   100¢ = ——— dollar
                                             10

7. 1 dime = $___._____   10¢ = ——— dollar
                                            10

8. 3 dimes = $___._____   30¢ = ——— dollar
                                            10

9. 5 dimes = $___._____   50¢ = ——— dollar
                                            10

10. 6 dimes = $___._____   60¢ = ——— dollar
                                            10

11. 4 quarters = $___._____   100¢ = ——— dollar
                                             100

12. 1 quarter = $___._____   25¢ = ——— dollar
                                            100

13. 2 quarters = $___._____   50¢ = ——— dollar
                                            100

14. 3 quarters = $___._____   75¢ = ——— dollar
                                            100

Solve. Give the total amount of money in fraction and decimal form.

15. 3 dimes and 8 pennies

16. 8 dimes and 23 pennies

17. 3 quarters 3 dimes and 5 pennies

18. 236 cents is what fraction of a dollar?

Solve. Express the answer as a decimal.

19. 2 dollars 17 pennies + 4 dollars 2 quarters

20. 3 dollars 8 dimes + 1 dollar 2 quarters 5 pennies

21. 9 dollars 9 dimes + 4 dollars 3 quarters 16 pennies

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Lesson 15:   Express money amounts given in various forms as decimal numbers.

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Name _____   Date _____

Solve. Give the total amount of money in fraction and decimal form.

1. 2 quarters and 3 dimes

2. 1 quarter 7 dimes and 23 pennies

Solve. Express the answer as a decimal.

3. 2 dollars 1 quarter 14 pennies + 3 dollars 2 quarters 3 dimes

Name _____   Date _____
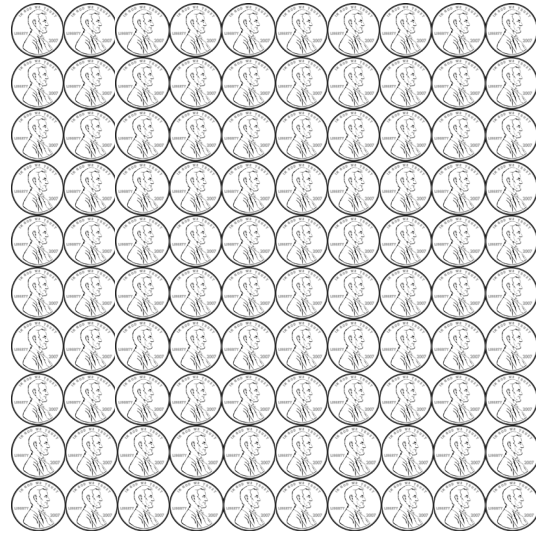
1. 100 pennies = $___._____   100¢ = $\frac{}{100}$ dollar

2. 1 penny = $___._____   1¢ = $\frac{}{100}$ dollar

3. 3 pennies = $___._____   3¢ = $\frac{}{100}$ dollar

4. 20 pennies = $___._____   20¢ = $\frac{}{100}$ dollar

5. 37 pennies = $___._____   37¢ = $\frac{}{100}$ dollar

6. 10 dimes = $___._____   100¢ = $\frac{}{10}$ dollar

7. 2 dimes = $___._____   20¢ = $\frac{}{10}$ dollar

8. 4 dimes = $___._____   40¢ = $\frac{}{10}$ dollar

9. 6 dimes = $___._____   60¢ = $\frac{}{10}$ dollar

10. 9 dimes = $___._____   90¢ = $\frac{}{10}$ dollar

11. 3 quarters = $___._____   75¢ = $\frac{}{100}$ dollar

12. 2 quarters = $___._____   50¢ = $\frac{}{100}$ dollar

13. 4 quarters = $___._____   100¢ = $\frac{}{100}$ dollar

14. 1 quarter = $___._____   25¢ = $\frac{}{100}$ dollar

Lesson 15:   Express money amounts given in various forms as decimal numbers.

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Solve. Give the total amount of money in fraction and decimal form.

15. 5 dimes and 8 pennies

16. 3 quarters and 13 pennies

17. 3 quarters 7 dimes and 16 pennies

18. 187 cents is what fraction of a dollar?

Solve. Express the answer in decimal form.

19. 1 dollar 2 dimes 13 pennies + 2 dollars 3 quarters

20. 2 dollars 6 dimes + 2 dollars 2 quarters 16 pennies

21. 8 dollars 8 dimes + 7 dollars 1 quarter 8 dimes

Lesson 16

Objective: Solve word problems involving money.

Suggested Lesson Structure

■ Fluency Practice   (12 minutes)
■ Concept Development   (38 minutes)
■ Student Debrief   (10 minutes)
  Total Time   **(60 minutes)**
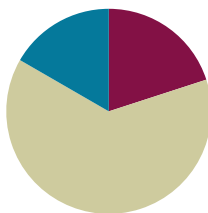
Fluency Practice (12 minutes)

▪ Sprint: Add Decimal Fractions **4.NF.5**   (9 minutes)
▪ State the Value of a Set of Coins **4.MD.2**   (3 minutes)

Sprint: Add Decimal Fractions (9 minutes)

Materials: (S) Add Decimal Fractions Sprint

Note: This Sprint reviews Lesson 13.

State the Value of a Set of Coins (3 minutes)

Materials: (S) Personal white board

Note: This fluency activity reviews Lesson 15.

T: (Write 2 quarters 4 dimes.) What is the value of 2 quarters and 4 dimes?
S: 90¢.
T: Write 90 cents as a fraction of a dollar.
S: (Write $\frac{90}{100}$ dollar.)
T: Write 90 cents in decimal form using the dollar symbol.
S: (Write $0.90.)
T: Write 130 cents in decimal form using the dollar symbol.
S: (Write $1.30.)
T: What is the value in cents of 3 quarters and 7 dimes?
S: 145¢.

T: Write 145 cents as a fraction of a dollar.

S: (Write $\frac{145}{100}$ dollar.)

T: Write 145 cents in decimal form using the dollar symbol.

S: (Write $1.45.)

Continue with the following possible sequence: 1 quarter 9 dimes 12 pennies, 3 quarters 5 dimes 20 pennies.

Concept Development (38 minutes)

Materials: (S) Problem Set

Suggested Delivery of Instruction for Solving This Lesson's Word Problems

Note: Lesson 15 closed with students finding sums of dollar and cents amounts in unit form. If necessary, begin this lesson with a short segment revisiting that process.

1. Model the problem.

Have two pairs of students model the problem at the board while the others work independently or in pairs at their seats. Review the following questions before beginning the first problem:

- Can you draw something?
- What can you draw?
- What conclusions can you make from your drawing?

As students work, circulate. Reiterate the questions above. After two minutes, have the two pairs of students share only their labeled diagrams. For about one minute, have the demonstrating students receive and respond to feedback and questions from their peers.

2. Calculate to solve and write a statement.

Give students two minutes to finish their work on that question, sharing their work and thinking with a peer. All should then write their equations and statements of the answer.

3. Assess the solution for reasonableness.

Give students one to two minutes to assess and explain the reasonableness of their solution.

> **NOTES ON**
> **MULTIPLE MEANS**
> **OF ENGAGEMENT:**
>
> Give everyone a fair chance to be successful by providing appropriate scaffolds. Demonstrating students may use translators, interpreters, or sentence frames to present and respond to feedback. Models shared may include concrete manipulatives, computer software, or other adaptive materials.
>
> If the pace of the lesson is a consideration, prepare presenters beforehand. The first problem may be most approachable for students working below grade level.

Problem 1

Miguel has 1 dollar bill, 2 dimes, and 7 pennies. John has 2 dollar bills, 3 quarters, and 9 pennies. How much money do the two boys have in all?

Students use their knowledge of mixed metric unit addition from Module 2 to add amounts of money. Each amount is expressed using the units of dollars and cents. Students know that 100 cents is equal to 1 dollar. Solution A shows a student decomposing 111 cents after finding the sum of the dollars and cents. Solution B shows a student decomposing Miguel's 27 cents to make 1 dollar before finding the total sum.

Problem 2

Suilin needs 7 dollars 13 cents to buy a book. In her wallet, she finds 3 dollar bills, 4 dimes, and 14 pennies. How much more money does Suilin need to buy the book?

Solution C

3 dollars 54 cents $\xrightarrow{+46¢}$ 4 dollars $\xrightarrow{+\$3}$ 7 dollars $\xrightarrow{+13¢}$ 7 dollars 13 cents

$3 + 46¢ + 13¢ = 59¢

= $3.59

Suilin needs $3.59 more to buy the book.

Students solve using unit form because they do not learn addition and subtraction of decimals until Grade 5. Solution A shows unbundling 1 dollar as 100 cents, making 113 cents to subtract 54 cents from. Solution B decomposed the cents in the subtrahend to more easily subtract from 1 dollar or 100 cents. Solution C adds up using the arrow way. Each solution shows conversion of the mixed unit into a decimal for dollars and cents.

Problem 3

Vanessa has 6 dimes and 2 pennies. Joachim has 1 dollar, 3 dimes, and 5 pennies. Jimmy has 5 dollars and 7 pennies. They want to put their money together to buy a game that costs $8.00. Do they have enough money to buy the game? If not, how much more money do they need?
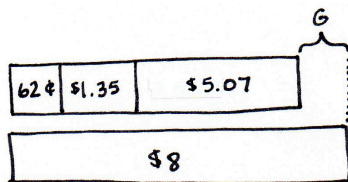
> **NOTES ON MULTIPLE MEANS FOR ACTION AND EXPRESSION:**
>
> Scaffold solving Problem 3 for students working below grade level by facilitating their management of information from the word problem. A labeled tape diagram, table, place value chart, or another organizational aid may help learners with cognitive disabilities keep information organized.

Solution A

62 cents + 1 dollar 35 cents + 5 dollars 7 cents

= 6 dollars 104 cents
= 7 dollars 4 cents
= $7.04

8 dollars − 7 dollars 4 cents
= 96 cents
G = $0.96

Solution B

8 dollars $\xrightarrow{-\$5}$ 3 dollars $\xrightarrow{-7¢}$ 2 dollars 93 cents

2 dollars 93 cents $\xrightarrow{-\$1}$ 1 dollar 93 cents $\xrightarrow{-35¢}$ 1 dollar 58 cents

158 cents $\xrightarrow{-62¢}$ 96 cents   G = $0.96

They don't have enough money. They need $0.96 more to buy the game.

In this multi-step problem, students may first find the sum of three money amounts and then subtract to find out how much more money they need, as shown in Solution A. Solution B shows the arrow way, subtracting each person's money one at a time.

Problem 4

A pen costs $2.29. A calculator costs 3 times as much as a pen. How much do a pen and a calculator cost together?

P | $2.29 |

C

$6.87

T
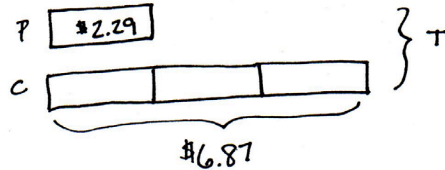
Solution A

$2.29 = 229 cents

229 cents
× 3
687 cents

6 dollars 87 cents + 2 dollars 29 cents

= 8 dollars 116 cents
          100 16

= 9 dollars 16 cents

T = $9.16

Solution B

$2.29 = 229 cents

229
× 4
916

916¢ = $9.16

T = $9.16

Solution C

230 cents
× 4
920 cents

T = $9.16

920 cents − 4 cents = 916 cents

A pen and a calculator cost $9.16 together.

In this multiplicative comparison word problem, students have to contemplate how to multiply money when they have not learned how to multiply with decimals. Solution A shows a student first solving for the cost of the calculator, then multiplying to find the total number of cents, and finally adding the cost of the pen after expressing the amount of each item as dollars and cents. Solution B is a more efficient method, solving for both items concurrently using cents. Solution C uses a compensation strategy to simplify the multiplication. Instead of a unit size of $2.29, the student adds 1 penny to each of the 4 units in the problem, finds 4 groups of $2.30, and then subtracts the 4 pennies that were added.

Problem 5

Krista has 7 dollars and 32 cents. Malory has 2 dollars and 4 cents. How much money does Krista need to give Malory so that each of them has the same amount of money?

Solution A
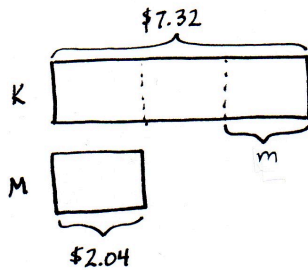
7 dollars 32 cents — 2 dollars 4 cents

= 5 dollars 28 cents

= 528 cents

$$2\overline{)528} = 264$$

264 cents = $2.64

M = $2.64

Solution B
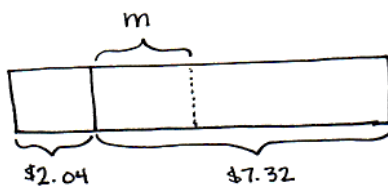
7 dollars 32 cents + 2 dollars 4 cents

= 9 dollars 36 cents

= 936 cents

$$2\overline{)936} = 468$$

468 cents = $4.68

2 dollars 4 cents $\xrightarrow{+64¢}$ 2 dollars 68 cents $\xrightarrow{+$2}$ 4 dollars 68 cents

m = $2.64

Malory needs $2.64 from Krista.

This challenging multi-step word problem requires students to divide money, similarly to Problem 4 with multiplication, by finding the total amount of cents since decimal division is a Grade 5 standard. Solution A divides the difference of money the girls have. Solution B divides the total amount of money, requiring an additional step either by finding how much more money Malory needs or subtracting from Krista's money.

Student Debrief (10 minutes)

Lesson Objective: Solve word problems involving money.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

Any combination of the questions below may be used to lead the discussion.

- Why does money relate so closely to our study of fractions and decimals?

- How could you use rounding to find the reasonableness of your answer to Problem 4? With your partner, estimate the cost of a pen and a calculator. Are your answers reasonable?

- In Problem 5, we saw two different tape diagrams drawn. How can the way you draw affect which strategy you choose to solve?

- Problem 5 can be challenging at first read. Think of an alternative scenario that may help a younger student solve a similar problem. (Consider using smaller numbers like 9 and 5 and a context like pieces of candy.)

Exit Ticket (3 minutes)

After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help with assessing students' understanding of the concepts that were presented in today's lesson and planning more effectively for future lessons. The questions may be read aloud to the students.

A

Number Correct: _____

Add Decimal Fractions

1.	$\frac{1}{10} =$	.	23.	$\frac{2}{10} =$	.
2.	$\frac{1}{100} =$	.	24.	$\frac{20}{100} =$	.
3.	$\frac{1}{10} + \frac{1}{100} =$	.	25.	$\frac{2}{10} + \frac{20}{100} =$	.
4.	$\frac{3}{10} =$	.	26.	$\frac{3}{10} =$	.
5.	$\frac{3}{100} =$	.	27.	$\frac{30}{100} =$	.
6.	$\frac{3}{10} + \frac{3}{100} =$	.	28.	$\frac{3}{10} + \frac{30}{100} =$	.
7.	$\frac{5}{10} =$	.	29.	$\frac{5}{10} + \frac{20}{100} =$	.
8.	$\frac{5}{100} =$	.	30.	$\frac{8}{10} + \frac{10}{100} =$	.
9.	$\frac{5}{10} + \frac{5}{100} =$	.	31.	$\frac{8}{10} + \frac{20}{100} =$	.
10.	$\frac{7}{10} =$	.	32.	$\frac{8}{10} + \frac{30}{100} =$	.
11.	$\frac{9}{100} =$	.	33.	$\frac{8}{10} + \frac{50}{100} =$	.
12.	$\frac{7}{10} + \frac{9}{100} =$	.	34.	$\frac{9}{10} + \frac{40}{100} =$	.
13.	$\frac{9}{100} + \frac{7}{10} =$	.	35.	$\frac{9}{10} + \frac{47}{100} =$	.
14.	$\frac{4}{10} =$	.	36.	$\frac{7}{10} + \frac{50}{100} =$	.
15.	$\frac{6}{100} =$	.	37.	$\frac{7}{10} + \frac{59}{100} =$	.
16.	$\frac{4}{10} + \frac{6}{100} =$	.	38.	$\frac{6}{10} + \frac{60}{100} =$	.
17.	$\frac{4}{100} + \frac{6}{10} =$	.	39.	$\frac{6}{10} + \frac{64}{100} =$	.
18.	$\frac{8}{10} + \frac{5}{100} =$	.	40.	$\frac{65}{100} + \frac{6}{10} =$	.
19.	$\frac{9}{10} + \frac{2}{100} =$	.	41.	$\frac{91}{100} + \frac{7}{10} =$	.
20.	$\frac{1}{100} + \frac{8}{10} =$	.	42.	$\frac{8}{10} + \frac{73}{100} =$	.
21.	$\frac{4}{100} + \frac{1}{10} =$	.	43.	$\frac{9}{10} + \frac{82}{100} =$	.
22.	$\frac{7}{100} + \frac{4}{10} =$	.	44.	$\frac{98}{100} + \frac{9}{10} =$	.

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B

Number Correct: _____

Improvement: _____

Add Decimal Fractions

1.	$\dfrac{2}{10} =$	.
2.	$\dfrac{2}{100} =$	.
3.	$\dfrac{2}{10} + \dfrac{2}{100} =$	.
4.	$\dfrac{4}{10} =$	.
5.	$\dfrac{4}{100} =$	.
6.	$\dfrac{4}{10} + \dfrac{4}{100} =$	.
7.	$\dfrac{6}{10} =$	.
8.	$\dfrac{6}{100} =$	.
9.	$\dfrac{6}{10} + \dfrac{6}{100} =$	.
10.	$\dfrac{4}{10} =$	.
11.	$\dfrac{8}{100} =$	.
12.	$\dfrac{4}{10} + \dfrac{8}{100} =$	.
13.	$\dfrac{8}{100} + \dfrac{4}{10} =$	.
14.	$\dfrac{5}{10} =$	.
15.	$\dfrac{7}{100} =$	.
16.	$\dfrac{5}{10} + \dfrac{7}{100} =$	.
17.	$\dfrac{7}{100} + \dfrac{5}{10} =$	.
18.	$\dfrac{9}{10} + \dfrac{6}{100} =$	.
19.	$\dfrac{8}{10} + \dfrac{3}{100} =$	.
20.	$\dfrac{1}{100} + \dfrac{7}{10} =$	.
21.	$\dfrac{3}{100} + \dfrac{1}{10} =$	.
22.	$\dfrac{8}{100} + \dfrac{3}{10} =$	.

23.	$\dfrac{1}{10} =$	.
24.	$\dfrac{10}{100} =$	.
25.	$\dfrac{1}{10} + \dfrac{10}{100} =$	.
26.	$\dfrac{4}{10} =$	.
27.	$\dfrac{40}{100} =$	.
28.	$\dfrac{4}{10} + \dfrac{40}{100} =$	.
29.	$\dfrac{5}{10} + \dfrac{30}{100} =$	.
30.	$\dfrac{7}{10} + \dfrac{20}{100} =$	.
31.	$\dfrac{7}{10} + \dfrac{30}{100} =$	.
32.	$\dfrac{7}{10} + \dfrac{40}{100} =$	.
33.	$\dfrac{7}{10} + \dfrac{60}{100} =$	.
34.	$\dfrac{9}{10} + \dfrac{30}{100} =$	.
35.	$\dfrac{9}{10} + \dfrac{37}{100} =$	.
36.	$\dfrac{8}{10} + \dfrac{40}{100} =$	.
37.	$\dfrac{8}{10} + \dfrac{49}{100} =$	.
38.	$\dfrac{7}{10} + \dfrac{70}{100} =$	.
39.	$\dfrac{7}{10} + \dfrac{76}{100} =$	.
40.	$\dfrac{78}{100} + \dfrac{7}{10} =$	.
41.	$\dfrac{81}{100} + \dfrac{7}{10} =$	.
42.	$\dfrac{9}{10} + \dfrac{73}{100} =$	.
43.	$\dfrac{9}{10} + \dfrac{84}{100} =$	.
44.	$\dfrac{84}{100} + \dfrac{8}{10} =$	.

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Name _____   Date _____

Use the RDW process to solve. Write your answer as a decimal.

1. Miguel has 1 dollar bill, 2 dimes, and 7 pennies. John has 2 dollar bills, 3 quarters, and 9 pennies. How much money do the two boys have in all?

2. Suilin needs 7 dollars 13 cents to buy a book. In her wallet, she finds 3 dollar bills, 4 dimes, and 14 pennies. How much more money does Suilin need to buy the book?

3. Vanessa has 6 dimes and 2 pennies. Joachim has 1 dollar, 3 dimes, and 5 pennies. Jimmy has 5 dollars and 7 pennies. They want to put their money together to buy a game that costs $8.00. Do they have enough money to buy the game? If not, how much more money do they need?

4. A pen costs $2.29. A calculator costs 3 times as much as a pen. How much do a pen and a calculator cost together?

5. Krista has 7 dollars and 32 cents. Malory has 2 dollars and 4 cents. How much money does Krista need to give Malory so that each of them has the same amount of money?

Name _____   Date _____

Use the RDW process to solve. Write your answer as a decimal.

David's mother told him that he could keep all the money he finds under the sofa cushions in their house. David finds 6 quarters, 4 dimes, and 26 pennies. How much money does David find altogether?

Name _____ Date _____

Use the RDW process to solve. Write your answer as a decimal.

1. Maria has 2 dollars, 3 dimes, and 4 pennies. Lisa has 1 dollar and 5 quarters. How much money do the two girls have in all?

2. Meiling needs 5 dollars 35 cents to buy a ticket to a show. In her wallet, she finds 2 dollar bills, 11 dimes, and 5 pennies. How much more money does Meiling need to buy the ticket?

3. Joe has 5 dimes and 4 pennies. Jamal has 2 dollars, 4 dimes, and 5 pennies. Jimmy has 6 dollars and 4 dimes. They want to put their money together to buy a book that costs $10.00. Do they have enough? If not, how much more do they need?

4. A package of mechanical pencils costs $4.99. A package of pens costs twice as much as a package of pencils. How much do a package of pens and a package of pencils cost together?

5. Carlos has 8 dollars and 48 cents. Alissa has 4 dollars and 14 cents. How much money does Carlos need to give Alissa so that each of them has the same amount of money?

Name _____   Date _____

1. Decompose each fraction into hundredths using area models. Then, write the equivalent number sentence using decimals.

   a. $\frac{8}{10} =$ ——

   b. $\frac{18}{10} =$ ——

   Decompose each fraction into hundredths. Then, write the equivalent number sentence for each part using decimals.

   c. $\frac{2}{10} =$ ——   d. $\frac{5}{10} =$ ——
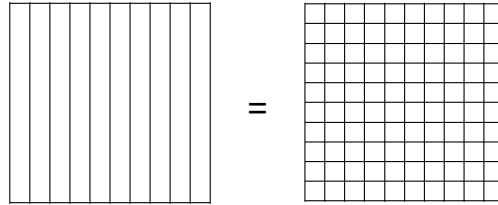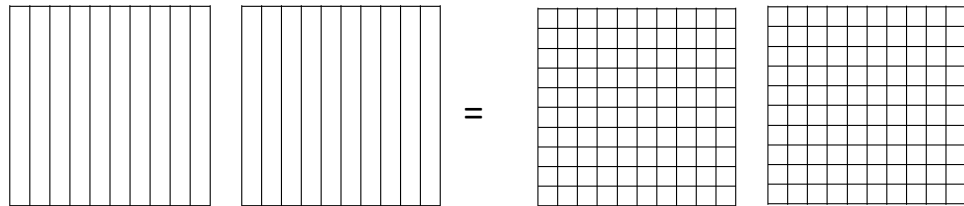
2. Several points are plotted on the number lines below. Identify the decimal number associated with each point.
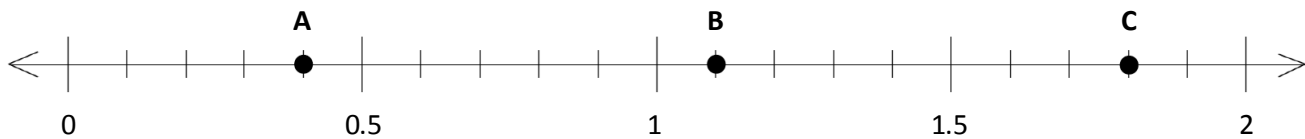
   A. _____   B. _____   C. _____
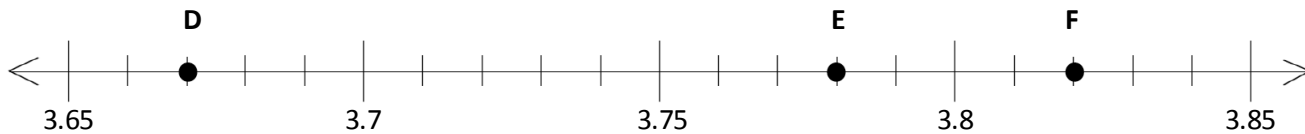
   D. _____   E. _____   F. _____

3. Use the symbols >, =, or < to compare the following. Justify your conclusions using pictures, numbers, or words.

   a.   0.02 $\bigcirc$ 0.22

   b.   0.6 $\bigcirc$ 0.60

   c.   17 tenths $\bigcirc$ 1.7

   d.   1.04 $\bigcirc$ $1\frac{4}{10}$

   e.   0.38 $\bigcirc$ $\frac{38}{10}$

   f.   4.05 $\bigcirc$ $4\frac{5}{100}$

   g.   3 tenths + 2 hundredths $\bigcirc$ 1 tenth + 13 hundredths

   h.   8 hundredths + 7 tenths $\bigcirc$ 6 tenths + 17 hundredths

4. Solve.

   a. Express your solution as a fraction of a meter.   0.3 m + 1.45 m

   b. Express your solution as a fraction of a liter.   1.7 L + 0.82 L

   c. Express your solution as a fraction of a dollar.   4 dimes 1 penny + 77 pennies

5. Solve.

   a. $\dfrac{7}{10} + \dfrac{8}{100}$

   b. $\dfrac{4}{10} + \dfrac{51}{100}$

   c. $\dfrac{5}{10} + \dfrac{68}{100}$

   d. $\dfrac{98}{100} + \dfrac{2}{10}$

   e. $\dfrac{12}{100} + \dfrac{12}{10}$

   f. $\dfrac{1}{10} + \dfrac{13}{100} + \dfrac{8}{10}$

6. Answer the following questions about a track meet.

    a. Jim and Joe ran in a relay race. Jim had a time of 9.8 seconds. Joe had a time of 10.32 seconds. Together, how long did it take them to complete the race? Record your answer as a decimal.

    b. The times of the 5 fastest runners were 7.11 seconds, 7.06 seconds, 7.6 seconds, 7.90 seconds, and 7.75 seconds. Locate these times on the number line. Record the times as decimals and fractions. One has been completed for you.

    7.11

    $7\frac{11}{100}$

    7   8
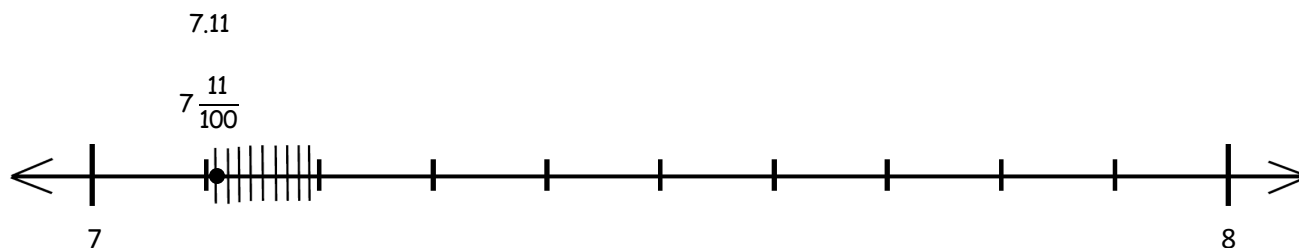
    c. Natalie threw a discus 32.04 meters. She threw 3.8 meters farther on her next throw. Write a statement to compare the two distances that Natalie threw the discus using >, <, or =.

d. At the concession stand, Marta spent 89 cents on a bottle of water and 5 dimes on a bag of chips. Shade the area models to represent the cost of each item.
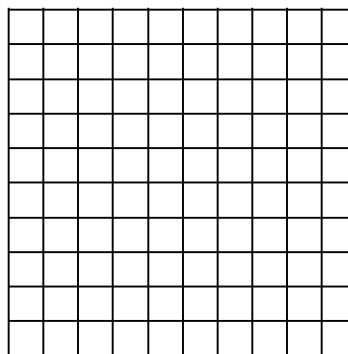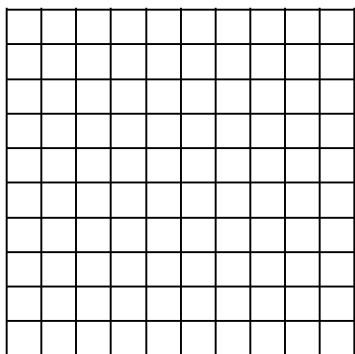
e. Write a number sentence in fraction form to find the total cost of a water bottle and a bag of chips. After solving, write the complete number sentence in decimal form.

f. Brian and Sonya each have a container. They mark their containers to show tenths. Brian and Sonya both fill their containers with 0.7 units of juice. However, Brian has more juice in his container. Explain how this is possible.

End-of-Module Assessment Task	**Topics A–E**
Standards Addressed	

Understand decimal notation for fractions, and compare decimal fractions.

4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.* (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.)

4.NF.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

Evaluating Student Learning Outcomes

A Progression Toward Mastery is provided to describe steps that illuminate the gradually increasing understandings that students develop *on their way to proficiency.* In this chart, this progress is presented from left (Step 1) to right (Step 4). The learning goal for students is to achieve Step 4 mastery. These steps are meant to help teachers and students identify and celebrate what the students CAN do now and what they need to work on next.

A Progression Toward Mastery

Assessment Task Item and Standards Assessed	STEP 1 Little evidence of reasoning without a correct answer. (1 Point)	STEP 2 Evidence of some reasoning without a correct answer. (2 Points)	STEP 3 Evidence of some reasoning with a correct answer or evidence of solid reasoning with an incorrect answer. (3 Points)	STEP 4 Evidence of solid reasoning with a correct answer. (4 Points)
1 4.NF.5 4.NF.6	Student answers fewer than two parts correctly.	Student answers two parts correctly.	Student correctly answers three of the four parts of the question, showing solid reasoning. OR Student answers all parts correctly without correctly modeling on the place value charts.	Student correctly uses the area models to represent: a. $\frac{8}{10} = \frac{80}{100}$; 0.8 = 0.80 b. $\frac{18}{10} = \frac{180}{100}$; 1.8 = 1.80 c. $\frac{2}{10} = \frac{20}{100}$; 0.2 = 0.20 d. $\frac{5}{10} = \frac{50}{100}$; 0.5 = 0.50
2 4.NF.6	Student correctly answers two or fewer parts of the question.	Student correctly answers three parts of the question.	Student correctly answers four or five parts of the question.	Student correctly answers: a. 0.4 b. 1.1 c. 1.8 d. 3.67 e. 3.78 f. 3.82
3 4.NF.6 4.NF.7	Student answers four or fewer parts of the question correctly with little to no reasoning.	Student correctly answers four or five parts of the question, providing evidence of some reasoning.	Student correctly answers six or seven parts of the question, with solid reasoning for each part correct. OR Student correctly solves all parts but does not provide solid reasoning for one or two parts.	Student correctly answers and reasons correctly using pictures, numbers, or words for each part: a. < b. = c. = d. < e. < f. = g. > h. >

A Progression Toward Mastery				
4 **4.NF.5**	Student correctly answers one or no parts.	Student correctly answers two parts of the question but does not include the units or provide ample evidence of reasoning.	Student correctly answers two of the three parts of the question, providing solid evidence or reasoning. OR Student solves all three parts correctly, providing only some or partially correct reasoning.	Student correctly answers: a. $1\frac{75}{100}$ meters b. $2\frac{52}{100}$ liters c. $1\frac{18}{100}$ dollars
5 **4.NF.5**	Student correctly answers two or fewer parts of the question.	Student correctly answers three or four of the six parts of the question.	Student correctly answers five of the six parts of the question.	Student correctly answers: a. $\frac{78}{100}$ b. $\frac{91}{100}$ c. $\frac{118}{100}$ or $1\frac{18}{100}$ d. $\frac{118}{100}$ or $1\frac{18}{100}$ e. $\frac{132}{100}$ or $1\frac{32}{100}$ f. $\frac{103}{100}$ or $1\frac{3}{100}$
6 **4.NF.5** **4.NF.6** **4.NF.7** **4.MD.2**	Student correctly answers fewer than three problems, providing little to no reasoning.	Student correctly answers three or four of the six problems, providing some reasoning.	Student correctly answers five of the six problems with solid reasoning. OR Student answers all six parts correctly but provides less than solid evidence in no more than two parts.	Student correctly: a. Answers 20.12 seconds. b. Plots the times on the number line and records each time as a decimal and fraction. c. Answers $32\frac{4}{100}$ m < $35\frac{84}{100}$ m; or 32.04 m < 35.84 m. d. Shades each area model representing each item.

A Progression Toward Mastery

| | | | | e. $\frac{89}{100} + \frac{50}{100} = \frac{139}{100} = 1\frac{39}{100}$; $0.89 + $0.50 = $1.39

f. Reasons that Brian's container of juice is larger, and, therefore, each tenth unit fills more juice than Sonya's container. Comparing is only valid when the unit whole is the same. The containers' unit wholes were of different sizes. |

Name ____Jack_____   Date _____

1. Decompose each fraction into hundredths using area models. Then, write the equivalent number sentence using decimals.

    a. $\frac{8}{10} = \frac{80}{100}$

    0.8 = 0.80
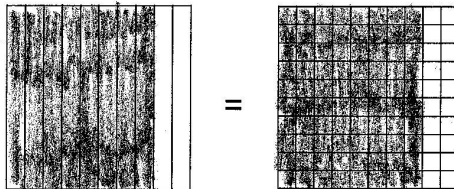
    b. $\frac{18}{10} = \frac{180}{100}$
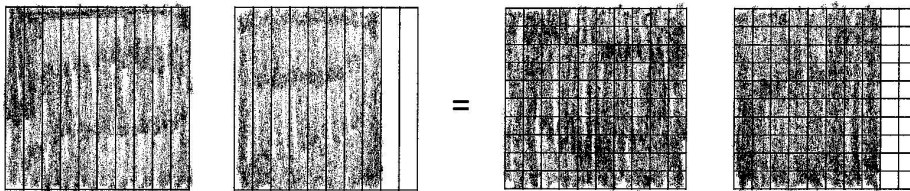
    1.8 = 1.80

    Decompose each fraction into hundredths. Then, write the equivalent number sentence for each part using decimals.

    c. $\frac{2}{10} = \frac{20}{100}$ 0.2 = 0.20   d. $\frac{5}{10} = \frac{50}{100}$ 0.5 = 0.50

2. Several points are plotted on the number lines below. Identify the decimal number associated with each point.
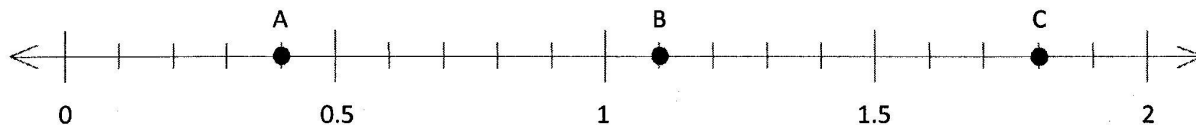
    A. __0.4__   B. __1.1__   C. __1.8__
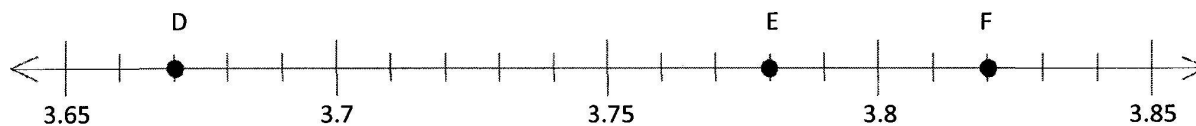
    D. __3.67__   E. __3.78__   F. __3.82__

3. Use the symbols >, =, or < to compare the following. Justify your conclusions using pictures, numbers, or words.

a. 0.02 $\boxed{<}$ 0.22

2 hundredths is less than 22 hundredths

b. 0.6 $\boxed{=}$ 0.60

$0.6 = \frac{6}{10}$ and $\frac{6}{10} = \frac{6 \times 10}{10 \times 10} = \frac{60}{100}$

$0.60 = \frac{60}{100}$ They are equal.

c. 17 tenths $\boxed{=}$ 1.7

$\frac{17}{10} = \frac{10}{10} + \frac{7}{10} = 1\frac{7}{10} = 1.7$

d. 1.04 $\boxed{<}$ $1\frac{4}{10}$

Hundredths are smaller than tenths, so 4 hundredths is less than 4 tenths. Since they both have one whole, $1.04 < 1\frac{4}{10}$.

e. 0.38 $\boxed{<}$ $\frac{38}{10}$

$\frac{38}{10} = 3\frac{8}{10}$

$\frac{30}{10}$ $\frac{8}{10}$

38 hundredths is less than 1. 38 tenths is greater than 1.

f. 4.05 $\boxed{=}$ $4\frac{5}{100}$

4.05 is 4 and 5 hundredths. That is the same as $4\frac{5}{100}$.

g. 3 tenths + 2 hundredths $\boxed{>}$ 1 tenth + 13 hundredths

$\frac{3}{10} + \frac{2}{100} = \frac{30}{100} + \frac{2}{100} = \frac{32}{100}$ $\frac{1}{10} + \frac{13}{100} = \frac{10}{100} + \frac{13}{100} = \frac{23}{100}$

32 hundredths is greater than 23 hundredths.

h. 8 hundredths + 7 tenths $\boxed{>}$ 6 tenths + 17 hundredths

$\frac{8}{100} + \frac{7}{10} = \frac{8}{100} + \frac{70}{100} = \frac{78}{100}$ $\frac{6}{10} + \frac{17}{100} = \frac{60}{100} + \frac{17}{100} = \frac{77}{100}$

78 hundredths is greater than 77 hundredths.

4. Solve.

   a. Express your solution as a fraction of a meter.   0.3 m + 1.45 m

$$\frac{3}{10}m + 1\frac{45}{100}m = \frac{30}{100}m + 1\frac{45}{100}m = 1\frac{75}{100}m$$

   b. Express your solution as a fraction of a liter.   1.7 L + 0.82 L

$$1\frac{7}{10}L + \frac{82}{100}L = 1\frac{70}{100}L + \frac{82}{100}L = 1\frac{152}{100}L = 2\frac{52}{100}L$$
$$\underset{\frac{100}{100}\quad\frac{52}{100}}{\diagup\quad\diagdown}$$

   c. Express your solution as a fraction of a dollar.   4 dimes 1 penny + 77 pennies

$$\frac{4}{10}\text{ dollar} + \frac{1}{100}\text{ dollar} + \frac{77}{100}\text{ dollar} = \frac{40}{100}\text{ dollar} + \frac{1}{100}\text{ dollar} + \frac{77}{100}\text{ dollar}$$
$$= \frac{118}{100}\text{ dollars} = 1\frac{18}{100}\text{ dollars}$$

5. Solve.

   a. $\frac{7}{10} + \frac{8}{100}$

$$\frac{70}{100} + \frac{8}{100} = \frac{78}{100}$$

   b. $\frac{4}{10} + \frac{51}{100}$

$$\frac{40}{100} + \frac{51}{100} = \frac{91}{100}$$

   c. $\frac{5}{10} + \frac{68}{100}$

$$\frac{50}{100} + \frac{68}{100} = \frac{118}{100} = 1\frac{18}{100}$$
$$\underset{\frac{100}{100}\quad\frac{18}{100}}{\diagup\quad\diagdown}$$

   d. $\frac{98}{100} + \frac{2}{10}$

$$\frac{98}{100} + \frac{20}{100} = \frac{118}{100} = 1\frac{18}{100}$$
$$\underset{\frac{100}{100}\quad\frac{18}{100}}{\diagup\quad\diagdown}$$

   e. $\frac{12}{100} + \frac{12}{10}$

$$\frac{12}{100} + \frac{120}{100} = \frac{132}{100} = 1\frac{32}{100}$$
$$\underset{\frac{100}{100}\quad\frac{32}{100}}{\diagup\quad\diagdown}$$

   f. $\frac{1}{10} + \frac{13}{100} + \frac{8}{10}$

$$\frac{10}{100} + \frac{13}{100} + \frac{80}{100} = \frac{103}{100} = 1\frac{3}{100}$$
$$\underset{\frac{100}{100}\quad\frac{3}{100}}{\diagup\quad\diagdown}$$

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6. Answer the following questions about a track meet.

a. Jim and Joe ran in a relay race. Jim had a time of 9.8 seconds. Joe had a time of 10.32 seconds. Together, how long did it take them to complete the race? Record your answer as a decimal.

$$9.8 = 9\frac{8}{10} = 9\frac{80}{100}$$

$$10.32 = 10\frac{32}{100}$$

$$9\frac{80}{100} + 10\frac{32}{100} = 19\frac{112}{100} = 20\frac{12}{100} = 20.12$$

$$\frac{100}{100} \quad \frac{12}{100}$$

It took them 20.12 seconds to complete the race.

b. The times of the 5 fastest runners were 7.11 seconds, 7.06 seconds, 7.6 seconds, 7.90 seconds, and 7.75 seconds. Locate these times on the number line. Record the times as decimals and fractions. One has been completed for you.
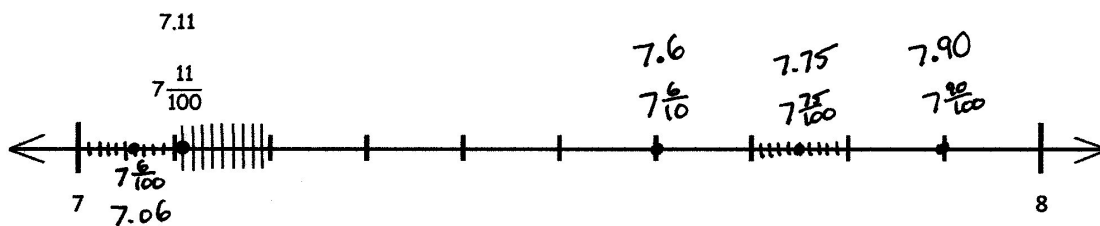
c. Natalie threw a discus 32.04 meters. She threw 3.8 meters farther on her next throw. Write a statement to compare the two distances that Natalie threw the discus using >, <, or =.

$$32.04 = 32\frac{4}{100}$$

$$3.8 = 3\frac{80}{100}$$

$$32\frac{4}{100} + 3\frac{80}{100} = 35\frac{84}{100} = 35.84$$

1ˢᵗ Throw 32.04 m   3.8m

2ⁿᵈ Throw

?
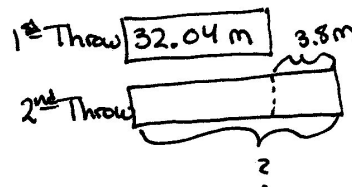
$$32.04\,m < 35.84\,m$$

d. At the concession stand, Marta spent 89 cents on a bottle of water and 5 dimes on a bag of chips.
Shade the area models to represent the cost of each item.

Bottle of Water

0.89

Bag of Chips

0.50
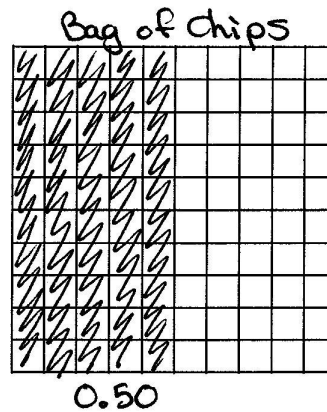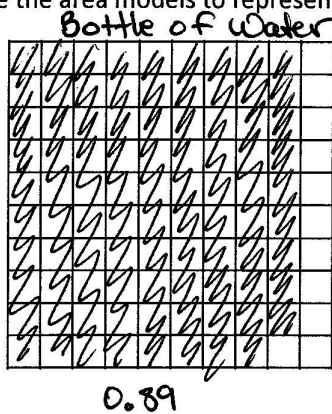
e. Write a number sentence in fraction form to find the total cost of a water bottle and a bag of chips.

After solving, write the complete number sentence in decimal form.

$$\frac{89}{100} + \frac{50}{100} = \frac{139}{100} = 1\frac{39}{100}$$

$$\frac{100}{100} \quad \frac{39}{100}$$

$$0.89 + 0.50 = 1.39$$
$$\$0.89 + \$0.50 = \$1.39$$

f. Brian and Sonya each have a container. They mark their containers to show tenths. Brian and Sonya
each fill their containers with 0.7 units of juice. However, Brian has more juice in his container.
Explain how this is possible.

It is possible that Brian has more juice in his container
because we don't know that Brian and Sonya's Containers
are the same size. If Brian's container is larger
than Sonya's, his tenths of a unit will be larger than
Sonya's and, therefore, he will have more juice.

Sonya   Brian
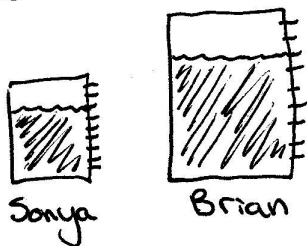
Answer Key

GRADE 4 • MODULE 6

Decimal Fractions

Lesson 1

Sprint

Side A

1.	2	12.	12	23.	5	34.	200
2.	3	13.	5	24.	85	35.	24
3.	4	14.	15	25.	185	36.	240
4.	8	15.	8	26.	7	37.	400
5.	5	16.	18	27.	27	38.	69
6.	9	17.	28	28.	427	39.	690
7.	7	18.	38	29.	9	40.	900
8.	6	19.	68	30.	59	41.	94
9.	1	20.	64	31.	759	42.	528
10.	10	21.	87	32.	12	43.	670
11.	2	22.	43	33.	120	44.	700

Side B

1.	1	12.	13	23.	4	34.	300
2.	2	13.	6	24.	84	35.	25
3.	3	14.	16	25.	184	36.	250
4.	7	15.	9	26.	8	37.	500
5.	4	16.	19	27.	28	38.	74
6.	8	17.	29	28.	428	39.	740
7.	6	18.	39	29.	6	40.	400
8.	5	19.	69	30.	56	41.	91
9.	9	20.	65	31.	756	42.	582
10.	10	21.	86	32.	13	43.	760
11.	3	22.	42	33.	130	44.	600

Problem Set

1. First 7 units shaded; 0.2, 0.3, …, 0.9;

   $\frac{0}{10}, \frac{2}{10}, \frac{3}{10}, …, \frac{10}{10}$; 0.7 circled

2. third container $\frac{9}{10}$ shaded

   $\frac{6}{10}$ = 0.6; $\frac{8}{10}$ = 0.8; $\frac{9}{10}$

3. $\frac{4}{10}$; 0.8; $\frac{2}{10}$ or 0.2

4. $\frac{5}{10}$; 0.5; 0.5 or $\frac{5}{10}$

5. a. $\frac{2}{10}$; 0.2

   b. $\frac{8}{10}$; 0.8

   c. $\frac{4}{10}$; 0.4

6. Lines drawn to matching values

Exit Ticket

1. a. $\frac{1}{10}$; 0.1

   b. $\frac{6}{10}$; 0.6

2. Lines drawn to matching values

Homework

1. First 4 units shaded; 0.2, 0.3, …, 0.9;

   $\frac{0}{10}, \frac{2}{10}, \frac{3}{10}, …, \frac{10}{10}$; 0.4 circled

2. $\frac{7}{10}$ = 0.7; $\frac{8}{10}$ = 0.8; $\frac{3}{10}$;

   third container $\frac{3}{10}$ shaded

3. $\frac{7}{10}$; 0.6

4. $\frac{5}{10}$; 0.5; 1

5. a. $\frac{6}{10}$; 0.6

   b. $\frac{7}{10}$; 0.7

   c. $\frac{2}{10}$; 0.2

6. Lines drawn to matching values

Lesson 2

Problem Set

1. Line segments drawn to given lengths
   a. 2.6 cm = $2\frac{6}{10}$ cm
   b. 3.4 cm = $3\frac{4}{10}$ cm
   c. 3.7 cm = $3\frac{7}{10}$ cm
   d. 4.2 cm = $4\frac{2}{10}$ cm
   e. 2.5 cm = $2\frac{5}{10}$ cm

2. Models shaded appropriately
   a. 2.6
   b. 4.2; $4 + \frac{2}{10} = 4 + 0.2 = 4.2$
   c. 3.4; $3 + \frac{4}{10} = 3 + 0.4 = 3.4$
   d. 2.5; $2 + \frac{5}{10} = 2 + 0.5 = 2.5$; 2.5
   e. 3.7; $3 + \frac{7}{10} = 3 + 0.7 = 3.7$; 1.3

Exit Ticket

1. Line segment drawn to given length; $4\frac{8}{10}$ cm

2. a. 3.7, $3\frac{7}{10}$; models shaded appropriately
   b. 2.4, $2\frac{4}{10}$; models shaded appropriately; 2.6

Homework

1. Line segments drawn to given lengths
   a. $2\frac{6}{10}$ cm
   b. $3\frac{5}{10}$ cm
   c. $1\frac{7}{10}$ cm
   d. $4\frac{3}{10}$ cm
   e. $2\frac{2}{10}$ cm

2. Models shaded appropriately
   a. 2.4
   b. 3.8; $3 + \frac{8}{10} = 3 + 0.8 = 3.8$
   c. 4.1; $4 + \frac{1}{10} = 4 + 0.1 = 4.1$
   d. 1.4; $1 + \frac{4}{10} = 1 + 0.4 = 1.4$; 3.6
   e. 3.3; $3 + \frac{3}{10} = 3 + 0.3 = 3.3$; 1.7

Lesson 3

Problem Set

1. a. 28; 2 ones and 8 tenths disks drawn; 2.8; 0.2

    b. 33; 3 ones and 3 tenths disks drawn; 3.3; 0.7

2. a. Answer provided

    b. Disks drawn appropriately

    $(1 \times 10) + (7 \times 1) + (5 \times \frac{1}{10}) = 17\frac{5}{10}$

    $(1 \times 10) + (7 \times 1) + (5 \times 0.1) = 17.5$

    c. Disks drawn appropriately

    $(2 \times 10) + (3 \times 1) + (2 \times \frac{1}{10}) = 23\frac{2}{10}$

    $(2 \times 10) + (3 \times 1) + (2 \times 0.1) = 23.2$

    d. Disks drawn appropriately

    $(7 \times 10) + (4 \times 1) + (7 \times \frac{1}{10}) = 74\frac{7}{10}$

    $(7 \times 10) + (4 \times 1) + (7 \times 0.1) = 74.7$

3. a. 3.9 plotted with endpoints 3 and 4; 3.9; $(3 \times 1) + (9 \times 0.1)$ or $(3 \times 1) + (9 \times \frac{1}{10})$

    b. 17.5; $17\frac{5}{10}$; $(1 \times 10) + (7 \times 1) + (5 \times \frac{1}{10})$ or $(1 \times 10) + (7 \times 1) + (5 \times 0.1)$; 0.5

    c. 74.7 plotted with endpoints 74 and 75; 74.7; $74\frac{7}{10}$; 0.3

    d. 22.2 plotted with endpoints 22 and 23; 22.2; $(2 \times 10) + (2 \times 1) + (2 \times 0.1)$ or $(2 \times 10) + (2 \times 1) + (2 \times \frac{1}{10})$; 0.8

    e. 80.8 plotted with endpoints 80 and 81; 80.8; $80\frac{8}{10}$; 0.2

Exit Ticket

1. 18; 1 one and 8 tenths disks drawn; 1.8; 0.2

2. a. 12.9 plotted with endpoints 12 and 13; 12.9; $(1 \times 10) + (2 \times 1) + (9 \times \frac{1}{10})$ or $(1 \times 10) + (2 \times 1) + (9 \times 0.1)$; 0.1

    b. 70.7 plotted with endpoints 70 and 71; $70\frac{7}{10}$; $(7 \times 10) + (7 \times \frac{1}{10})$ or $(7 \times 10) + (7 \times 0.1)$; 0.3

Homework

1. a. 14; 1 one and 4 tenths disks drawn; 1.4; 0.6

    b. 25; 2 ones and 5 tenths disks drawn; 2.5; 0.5

2. a. Answer provided

    b. Disks drawn appropriately

    $(5 \times 10) + (3 \times 1) + (7 \times \frac{1}{10}) = 53\frac{7}{10}$

    $(5 \times 10) + (3 \times 1) + (7 \times 0.1) = 53.7$

    c. Disks drawn appropriately

    $(3 \times 10) + (2 \times 1) + (3 \times \frac{1}{10}) = 32\frac{3}{10}$

    $(3 \times 10) + (2 \times 1) + (3 \times 0.1) = 32.3$

    d. Disks drawn appropriately

    $(8 \times 10) + (4 \times 1) + (8 \times \frac{1}{10}) = 84\frac{8}{10}$

    $(8 \times 10) + (4 \times 1) + (8 \times 0.1) = 84.8$

3. a. 4.6 plotted with endpoints 4 and 5; 4.6; $(4 \times 1) + (6 \times \frac{1}{10})$ or $(4 \times 1) + (6 \times 0.1)$; 0.4

    b. 24.5, $24\frac{5}{10}$; $(2 \times 10) + (4 \times 1) + (5 \times \frac{1}{10})$ or $(2 \times 10) + (4 \times 1) + (5 \times 0.1)$; 0 .5

    c. 63.6 plotted with endpoints 63 and 64; 63.6; $63\frac{6}{10}$; 0.4

    d. 71.3 plotted with endpoints 71 and 72; 71.3; $(7 \times 10) + (1 \times 1) + (3 \times \frac{1}{10})$ or

    $(7 \times 10) + (1 \times 1) + (3 \times 0.1)$; 0.7

    e. 90.9 plotted with endpoints 90 and 91; 90.9; $90\frac{9}{10}$; 0.1

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Lesson 4

Sprint

Side A

1.	0.2	12.	8	23.	10	34.	4.3
2.	0.3	13.	0.7	24.	20	35.	2.0
3.	0.4	14.	4	25.	50	36.	18
4.	0.8	15.	0.9	26.	40	37.	3.4
5.	0.6	16.	1.0	27.	41	38.	5.0
6.	1	17.	1.1	28.	42	39.	47
7.	2	18.	1.2	29.	46	40.	2.8
8.	3	19.	1.5	30.	26	41.	3.0
9.	7	20.	2.5	31.	36	42.	32
10.	5	21.	4.5	32.	34	43.	2.0
11.	0.5	22.	3.8	33.	23	44.	21

Side B

1.	0.1	12.	9	23.	10	34.	3.4
2.	0.2	13.	0.6	24.	20	35.	5.0
3.	0.3	14.	5	25.	40	36.	17
4.	0.7	15.	0.9	26.	30	37.	4.3
5.	0.5	16.	1.0	27.	31	38.	2.0
6.	2	17.	1.1	28.	32	39.	46
7.	3	18.	1.2	29.	36	40.	2.4
8.	4	19.	1.7	30.	16	41.	4.0
9.	8	20.	2.7	31.	26	42.	23
10.	6	21.	4.7	32.	42	43.	3.0
11.	0.4	22.	3.4	33.	25	44.	41

Problem Set

1. a. 10 cm

   b. $\frac{1}{100}$ m

   c. $\frac{1}{10}$ m or $\frac{10}{100}$ m

   d. 0.1 m or 0.10 m

   e. $\frac{1}{10}$ m or $\frac{10}{100}$ m

2. a. 10

   b. 10

   c. 100

3. a. Answer provided

   b. $\frac{2}{10}$ m + $\frac{4}{100}$ m = $\frac{24}{100}$ m = 0.24 m; number bond showing $\frac{2}{10}$ m and $\frac{4}{100}$ m is 0.24 m

   c. $\frac{3}{10}$ m + $\frac{8}{100}$ m = $\frac{38}{100}$ m = 0.38 m; number bond showing $\frac{3}{10}$ m and $\frac{8}{100}$ m is 0.38 m

4. a. Shaded appropriately; 0.8 m

   b. Shaded appropriately; 0.07 m

   c. Shaded appropriately; 0.19 m

5. a. Number bond showing $\frac{1}{10}$ m and $\frac{9}{100}$ m is 0.19 m

   b. Number bond showing $\frac{2}{10}$ m and $\frac{8}{100}$ m is 0.28 m

   c. Number bond showing $\frac{7}{10}$ and $\frac{7}{100}$ is 0.77

   d. Number bond showing $\frac{9}{10}$ and $\frac{4}{100}$ is 0.94

Exit Ticket

1. Shaded appropriately; 0.6 m

2. a. Number bond showing $\frac{6}{10}$ m and $\frac{2}{100}$ m is 0.62 m

   b. Number bond showing $\frac{2}{10}$ and $\frac{7}{100}$ is 0.27

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Homework

1. a. 30 cm

   b. $\frac{3}{100}$ m

   c. $\frac{3}{10}$ m or $\frac{30}{100}$ m

   d. 0.3 m or 0.30 m

   e. $\frac{3}{10}$ m or $\frac{30}{100}$ m

2. a. 50

   b. 50

   c. 100

3. a. Answer provided

   b. $\frac{3}{10}$ m + $\frac{8}{100}$ m = $\frac{38}{100}$ m = 0.38 m; number bond showing $\frac{3}{10}$ m and $\frac{8}{100}$ m is 0.38 m

   c. $\frac{4}{10}$ m + $\frac{6}{100}$ m = $\frac{46}{100}$ m = 0.46 m; number bond showing $\frac{4}{10}$ m and $\frac{6}{100}$ m is 0.46 m

4. a. Shaded appropriately; 0.9 m

   b. Shaded appropriately; 0.15 m

   c. Shaded appropriately; 0.41 m

5. a. Number bond showing $\frac{2}{10}$ m and $\frac{3}{100}$ m is 0.23 m

   b. Number bond showing $\frac{3}{10}$ m and $\frac{8}{100}$ m is 0.38 m

   c. Number bond showing $\frac{8}{10}$ and $\frac{2}{100}$ is 0.82

   d. Number bond showing $\frac{7}{10}$ and $\frac{6}{100}$ is 0.76

Lesson 5

Problem Set

1. a. 10, 10, 30; models shaded appropriately; 0.30

    b. 10, 10, 5; models shaded appropriately; 0.5

2. a. 3, 7; $\frac{37}{100}$; 0.37; model shaded appropriately

    b. 7, 5; $\frac{75}{100}$; 0.75; model shaded appropriately

3. a. 10 disks circled; 12, 1, 2; number bond showing $\frac{1}{10}$ and $\frac{2}{100}$ is 0.12

    b. 2 groups of 10 disks circled; 27, 2, 7; number bond showing $\frac{2}{10}$ and $\frac{7}{100}$ is 0.27

4. a. 03; 3; 3 (0.01) disks drawn

    b. 15; 1, 5; 1 (0.1) disk and 5 (0.01) disks drawn

    c. $\frac{72}{100}$; 72; 7 (0.1) disks and 2 (0.01) disks drawn

    d. $\frac{8}{10}$; 8; 8 (0.1) disks drawn

    e. $\frac{72}{100}$; 72; 7 (0.1) disks and 2 (0.01) disks drawn

    f. $\frac{80}{100}$; 80; 8 (0.1) disks drawn

Exit Ticket

1. 07; 7; 7 (0.01) disks drawn

2. 34; 3, 4; 3 (0.1) disks and 4 (0.01) disks drawn

Homework

1. a. 10, 10, 40; model shaded appropriately; 0.4 or 0.40

    b. 10, 10, 6; model shaded appropriately; 0.6

2. a. 3, 6; 0.36; $\frac{36}{100}$; model shaded appropriately

    b. 8, 2; 0.82; $\frac{82}{100}$; model shaded appropriately

3. a. 1 group of 10 disks circled; 14, 1, 4; number bond showing $\frac{1}{10}$ and $\frac{4}{100}$ is 0.14

    b. 2 groups of 10 disks circled; 24, 2, 4; number bond showing $\frac{2}{10}$ and $\frac{4}{100}$ is 0.24

4. a. 04; 4; 4 (0.01) disks drawn

    b. 13; 1, 3; 1 (0.1) disk and 3 (0.01) disks drawn

    c. $\frac{41}{100}$; 41; 4 (0.1) disks and 1 (0.01) disk drawn

    d. $\frac{9}{10}$; 9; 9 (0.1) disks drawn

    e. $\frac{63}{100}$; 63; 6 (0.1) disks and 3 (0.01) disks drawn

    f. $\frac{90}{100}$; 90; 9 (0.1) disks drawn

Lesson 6

Problem Set

1. a. 1.15, model shaded appropriately; point plotted accurately on number line

    b. 2.47, model shaded appropriately; point plotted accurately on number line

2. a. Point plotted accurately on number line

    b. Point plotted accurately on number line

3. a. $1\frac{2}{100}$; 1.02   d. $2\frac{27}{100}$; 2.27

    b. $1\frac{17}{100}$; 1.17   e. $4\frac{58}{100}$; 4.58

    c. $2\frac{8}{100}$; 2.08   f. $7\frac{70}{100}$; 7.70

4. Lines drawn to matching values

Exit Ticket

1. a. 7.20 plotted accurately on number line

    b. 1.75 plotted accurately on number line

2. a. $8\frac{24}{100}$; 8.24

    b. $2\frac{6}{100}$; 2.06

Homework

1. a. 2.35; model shaded appropriately; point plotted accurately on number line

    b. 3.17; model shaded appropriately; point plotted accurately on number line

2. a. 5.9 plotted accurately on number line

    b. 3.25 plotted accurately on number line

3. a. $2\frac{2}{100}$; 2.02   d. $1\frac{18}{100}$; 1.18

    b. $2\frac{16}{100}$; 2.16   e. $9\frac{62}{100}$; 9.62

    c. $3\frac{7}{100}$; 3.07   f. $6\frac{20}{100}$; 6.20

4. Lines drawn to matching values

Lesson 7

Problem Set

1. a. $20 + 0.5 + 0.03 = 20.53$

   b. $500 + 0.04 = 500.04$

2. a. 4; 4 hundreds   e. 5; 5 hundreds

   b. 1; 1 ten   f. 3; 3 tens

   c. 8; 8 tenths   g. 1; 1 tenth

   d. 3; 3 hundredths   h. 6; 6 hundredths

3.

$21\frac{4}{10}$ $(2 \times 10) + (1 \times 1) + (4 \times \frac{1}{10})$ $(2 \times 10) + (1 \times 1) + (4 \times 0.1)$

$20 + 1 + \frac{4}{10}$ $20 + 1 + 0.4$

$38\frac{9}{100}$ $(3 \times 10) + (8 \times 1) + (9 \times \frac{1}{100})$ $(3 \times 10) + (8 \times 1) + (9 \times 0.01)$

$30 + 8 + \frac{9}{100}$ $30 + 8 + 0.09$

$50\frac{2}{10}$ $(5 \times 10) + (2 \times \frac{1}{10})$ $(5 \times 10) + (2 \times 0.1)$

$50 + \frac{2}{10}$ $50 + 0.2$

$301\frac{7}{100}$ $(3 \times 100) + (1 \times 1) + (7 \times \frac{1}{100})$ $(3 \times 100) + (1 \times 1) + (7 \times 0.01)$

$300 + 1 + \frac{7}{100}$ $300 + 1 + 0.07$

$620\frac{80}{100}$ $(6 \times 100) + (2 \times 10) + (80 \times \frac{1}{100})$ $(6 \times 100) + (2 \times 10) + (80 \times 0.01)$

$600 + 20 + \frac{80}{100}$ $600 + 20 + 0.80$

$800\frac{8}{100}$ $(8 \times 100) + (8 \times \frac{1}{100})$ $(8 \times 100) + (8 \times 0.01)$

$800 + \frac{8}{100}$ $800 + 0.08$

Exit Ticket

1. a. 8; 8 hundreds

    b. 2; 2 tens

    c. 6; 6 tenths

    d. 4; 4 hundredths

2. $(4 \times 100) + (2 \times 10) + (2 \times 1) + (8 \times \frac{1}{100})$ $(4 \times 100) + (2 \times 10) + (2 \times 1) + (8 \times 0.01)$ 422.08

    $300 \frac{92}{100}$ $(3 \times 100) + (9 \times 0.1) + (2 \times 0.01)$ 300.92

Homework

1. a. 30 + 0.4 + 0.02 = 30.42

    b. 400 + 0.03 = 400.03

2. a. 8, 8 hundreds   e. 3, 3 hundreds

    b. 2, 2 tens   f. 4, 4 tens

    c. 6, 6 tenths   g. 1, 1 tenth

    d. 4, 4 hundredths   h. 9, 9 hundredths

3.
$25\frac{3}{10}$	$(2 \times 10) + (5 \times 1) + (3 \times \frac{1}{10})$ $20 + 5 + \frac{3}{10}$	$(2 \times 10) + (5 \times 1) + (3 \times 0.1)$ $20 + 5 + 0.3$
$39\frac{7}{100}$	$(3 \times 10) + (9 \times 1) + (7 \times \frac{1}{100})$ $30 + 9 + \frac{7}{100}$	$(3 \times 10) + (9 \times 1) + (7 \times 0.01)$ $30 + 9 + 0.07$
$40\frac{6}{10}$	$(4 \times 10) + (6 \times \frac{1}{10})$ $40 + \frac{6}{10}$	$(4 \times 10) + (6 \times 0.1)$ $40 + 0.6$
$208\frac{90}{100}$	$(2 \times 100) + (8 \times 1) + (90 \times \frac{1}{100})$ $200 + 8 + \frac{90}{100}$	$(2 \times 100) + (8 \times 1) + (9 \times 0.1)$ $200 + 8 + 0.9$
$510\frac{7}{100}$	$(5 \times 100) + (1 \times 10) + (7 \times \frac{1}{100})$ $500 + 10 + \frac{7}{100}$	$(5 \times 100) + (1 \times 10) + (7 \times 0.01)$ $500 + 10 + 0.07$
$900\frac{9}{100}$	$(9 \times 100) + (9 \times \frac{1}{100})$ $900 + \frac{9}{100}$	$(9 \times 100) + (9 \times 0.01)$ $900 + 0.09$

Lesson 8

Sprint

Side A

1.	0.3	12.	$\frac{6}{10}$	23.	2.16	34.	30.72
2.	0.03	13.	0.07	24.	2.16	35.	2.35
3.	0.23	14.	$\frac{2}{100}$	25.	3.16	36.	4.53
4.	1.23	15.	0.09	26.	3.14	37.	4.53
5.	4.23	16.	0.10	27.	3.54	38.	4.53
6.	$\frac{7}{100}$	17.	0.12	28.	2.38	39.	20.81
7.	$1\frac{7}{100}$	18.	0.12	29.	2.08	40.	4.29
8.	$\frac{7}{10}$	19.	0.13	30.	1.3	41.	2.74
9.	$1\frac{7}{10}$	20.	0.14	31.	10.3	42.	8.62
10.	$1\frac{74}{100}$	21.	0.19	32.	1.46	43.	8.95
11.	0.04	22.	3.19	33.	10.46	44.	10.94

Side B

1.	0.1	12.	$\frac{9}{10}$	23.	2.14	34.	30.96
2.	0.2	13.	0.6	24.	2.14	35.	2.57
3.	0.3	14.	$\frac{5}{10}$	25.	3.14	36.	4.75
4.	0.7	15.	0.9	26.	3.16	37.	4.75
5.	0.5	16.	1.0	27.	3.56	38.	4.75
6.	$\frac{2}{10}$	17.	1.1	28.	2.49	39.	20.61
7.	$\frac{3}{10}$	18.	1.2	29.	2.06	40.	6.47
8.	$\frac{4}{10}$	19.	1.7	30.	1.5	41.	2.96
9.	$\frac{8}{10}$	20.	2.7	31.	10.5	42.	6.84
10.	$\frac{6}{10}$	21.	4.7	32.	1.24	43.	8.73
11.	0.4	22.	3.4	33.	10.24	44.	10.76

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Problem Set

1. a. Area model accurately shaded;

    25, 2, 5, 2.5

    b. Explanations will vary.

2. 20; disks drawn to model number

    20; disks drawn to model number

    13; disks drawn to model number

    23; disks drawn to model number

3. a. 10

    b. 20

    c. 17

    d. 29

    e. 107

    f. 209

4. a. 100

    b. 200

    c. 170

    d. 290

    e. 1070

    f. 2090

5. $4\frac{2}{10}$; 42 tenths, $\frac{42}{10}$; 420 hundredths, $\frac{420}{100}$

    $8\frac{4}{10}$; 84 tenths, $\frac{84}{10}$; 840 hundredths, $\frac{840}{100}$

    $10\frac{2}{10}$; 102 tenths, $\frac{102}{10}$; 1020 hundredths, $\frac{1020}{100}$

    $75\frac{5}{10}$; 755 tenths, $\frac{755}{10}$; 7550 hundredths, $\frac{7550}{100}$

Exit Ticket

1. a. 32; disks drawn to model number

    b. 320

2. a. 26

    b. 610

Homework

1. a. Area model accurately shaded;

    22, 2, 2, 2.2

    b. Explanations will vary.

2. 30; disks drawn to model number

    30; disks drawn to model number

    23; disks drawn to model number

    33; disks drawn to model number

3. a. 10

    b. 20

    c. 13

    d. 26

    e. 103

    f. 206

4. a. 100

    b. 200

    c. 130

    d. 260

    e. 1030

    f. 2060

5. $5\frac{3}{10}$; 53 tenths, $\frac{53}{10}$; 530 hundredths, $\frac{530}{100}$

    $9\frac{7}{10}$; 97 tenths, $\frac{97}{10}$; 970 hundredths, $\frac{970}{100}$

    $10\frac{9}{10}$; 109 tenths, $\frac{109}{10}$; 1090 hundredths, $\frac{1090}{100}$

    $68\frac{5}{10}$; 685 tenths, $\frac{685}{10}$; 6850 hundredths, $\frac{6850}{100}$

Lesson 9

Problem Set

1. a. 0.3 m, 0.27 m; 0.3 meters is longer than 0.27 meters, or 0.27 meters is shorter than 0.3 meters.
    b. 0.35 m, 0.4 m; 0.4 meters is longer than 0.35 meters, or 0.35 meters is shorter than 0.4 meters.
    c. 0.27 m, 0.3 m, 0.35 m, 0.4 m

2. a. Bananas and grapes (0.6 kg and 0.61 kg) crossed off
    b. 0.2; 0.12; 0.6; 0.61
    c. Heavier than; lighter than

3. 0.6; 0.3; 0.9; 0.97; 0.19; 0.48
    a. >
    b. <
    c. >
    d. 0.19 L, 0.3 L, 0.48 L, 0.6 L, 0.9 L, 0.97 L

Exit Ticket

1. a. 0.54 m; 0.5 m; 0.47 m
    b. 0.54 m, 0.5 m, 0.47 m

2. a. >
    b. <
    c. <

Homework

1. a. 0.68 m, 0.7 m; 0.7 meters is longer than 0.68 meters, or 0.68 meters is shorter than 0.7 meters.
    b. 0.5 m, 0.44 m; 0.5 meters is longer than 0.44 meters, or 0.44 meters is shorter than 0.5 meters.
    c. 0.44 m, 0.5 m, 0.68 m, 0.7 m

2. a. Basketball and soccer ball (0.62 kg and 0.43 kg) crossed off
    b. 0.15; 0.25; 0.62; 0.43
    c. Heavier than; lighter than

3. 0.7; 0.62; 0.28; 0.4; 0.85; 0.2
    a. >
    b. <
    c. <
    d. 0.2 L, 0.28 L, 0.4 L, 0.62 L, 0.7 L, 0.85 L

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Lesson 10

Problem Set

1. a. <; models shaded appropriately
    b. >; models shaded appropriately
    c. <; models shaded appropriately
    d. =; models shaded appropriately

2. a. <; points plotted and labeled accurately
    b. <; points plotted and labeled accurately

3. a. <
    b. >
    c. <
    d. <
    e. =
    f. >

4. a. =
    b. <
    c. <
    d. <
    e. <
    f. <

Exit Ticket

1. =; Ryan and Jessie are wrong, answers may vary; models shaded appropriately

2. a. >
    b. >
    c. =

Homework

1. a. <; models shaded appropriately
    b. >; models shaded appropriately
    c. >; models shaded appropriately
    d. <; models shaded appropriately

2. a. >; points plotted and labeled accurately
    b. <; points plotted and labeled accurately

3. a. >
    b. <
    c. >
    d. <
    e. =
    f. <

4. a. =
    b. >
    c. >
    d. <
    e. =
    f. >

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Lesson 11

Problem Set

1. a. Points plotted accurately in following order: $\frac{1}{10}$, $\frac{12}{100}$, 0.2, 0.21, $\frac{32}{100}$, 0.33

    b. Points plotted accurately in following order: 3.62, $\frac{364}{100}$, 3.7, $\frac{38}{10}$, $3\frac{85}{100}$

    c. Points plotted accurately in following order: $\frac{62}{10}$, $\frac{628}{100}$, $6\frac{3}{10}$, 6.31, 6.40, 6.43

2. a. 2.72 > 2.71 > 2.7 > 2.27 > 2.07 > 0.27

    b. 13.2 > 13.02 > 12.3 > 12.2 > 1.34

    c. 75 > 7.5 > 7.4 > 7.34 > 7.3 > 7.2

3. Rhonda

4. December; March

Exit Ticket

1. Points plotted accurately in following order: 1.02, 1.1, 1.11, 1.20, 1.29, 1.3

2. 6.56 > 6.5 > 6.15 > 6.05 > 5.6 > 5.16

Homework

1. a. Points plotted accurately in following order: 0.5, 0.53, 0.6, 0.67, 0.76, 0.79

    b. Points plotted accurately in following order: 8.1, 8.15, 8.2, 8.27, 8.32

    c. Points plotted accurately in following order: 13, 13.03, 13.12, 13.21, 13.3

2. a. 4.43 > 4.33 > 4.31 > 4.30 > 4.03 > 0.34

    b. 17.55 > 17.5 > 17.05 > 15.75 > 15.71 > 15.7

    c. 81 > 9.8 > 8.9 > 8.19 > 8.1 > 8.09

3. Jenna

4. Monday; Wednesday

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Lesson 12

Problem Set

1. a. 15
    b. 21; disks modeled appropriately in chart
    c. 22; disks modeled appropriately in chart

2. a. 10, 13
    b. 50, 12, 62
    c. 70, 27, 97
    d. 37, 70, 107

3. a. 0.28
    b. 0.53
    c. 0.99
    d. 1.0 or 1

4. a. 1.32
    b. 1.2
    c. 1.48
    d. 1.57

5. 1.03 L

Exit Ticket

1. 19; disks modeled appropriately in chart

2. 1.13

Homework

1. a. 18
    b. 23; disks modeled appropriately in chart
    c. 24; disks modeled appropriately in chart

2. a. 10, 12
    b. 40, 11, 51
    c. 80, 25, 105
    d. 43, 60, 103

3. a. 0.37
    b. 0.66
    c. 0.90 or 0.9
    d. 1.0 or 1

4. a. 1.03
    b. 1.07
    c. 1.18
    d. 1.68

5. 1.48 inches

Lesson 13

Problem Set

1.
   a. $2\frac{13}{100}$; 2.13

   b. $7\frac{13}{100}$; 2.1 + 5.03 = 7.13

   c. $3\frac{24}{100} + \frac{70}{100} = 3\frac{94}{100}$; 3.24 + 0.7 = 3.94

   d. $3\frac{24}{100} + 8\frac{70}{100} = 11\frac{94}{100}$; 3.24 + 8.7 = 11.94

2.
   a. 8; 6.9 + 1.1 = 8.0 or 8

   b. $12\frac{35}{100}$; 9.9 + 2.45 = 12.35

   c. $11\frac{30}{100}$; 2.4 + 8.90 = 11.3

   d. $14\frac{7}{100}$; 6.37 + 7.7 = 14.07

3.
   a. $6\frac{4}{10} + 5\frac{3}{10} = 11\frac{7}{10}$; 6.4 + 5.3 = 11.7

   b. $6\frac{62}{100} + 2\frac{98}{100} = 9\frac{60}{100}$; 6.62 + 2.98 = 9.6

   c. $2\frac{10}{100} + \frac{94}{100} = 3\frac{4}{100}$; 2.10 + 0.94 = 3.04

   d. $2\frac{10}{100} + 5\frac{94}{100} = 8\frac{4}{100}$; 2.10 + 5.94 = 8.04

   e. $5\frac{70}{100} + 4\frac{92}{100} = 10\frac{62}{100}$; 5.70 + 4.92 = 10.62

   f. $5\frac{68}{100} + 4\frac{90}{100} = 10\frac{58}{100}$; 5.68 + 4.90 = 10.58

   g. $4\frac{80}{100} + 3\frac{27}{100} = 8\frac{7}{100}$; 4.8 + 3.27 = 8.07

   h. $17\frac{60}{100} + 3\frac{59}{100} = 21\frac{19}{100}$; 17.6 + 3.59 = 21.19

Exit Ticket

1. $7\frac{30}{100} + \frac{95}{100} = 8\frac{25}{100}$; 7.3 + 0.95 = 8.25

2. $8\frac{29}{100} + 5\frac{90}{100} = 14\frac{19}{100}$; 8.29 + 5.9 = 14.19

Homework

1.
   a. $5\frac{27}{100}$; 5.27

   b. $8\frac{27}{100}$; 5.2 + 3.07 = 8.27

   c. $6\frac{50}{100} + \frac{1}{100} = 6\frac{51}{100}$; 6.5 + 0.01 = 6.51

   d. $6\frac{50}{100} + 7\frac{1}{100} = 13\frac{51}{100}$; 6.5 + 7.01 = 13.51

2.
   a. 10; 4.9 + 5.1 = 10.0 or 10

   b. $11\frac{35}{100}$; 8.7 + 2.65 = 11.35

   c. $14\frac{17}{100}$; 7.3 + 6.87 = 14.17

   d. $13\frac{28}{100}$; 5.48 + 7.8 = 13.28

3.
   a. $2\frac{97}{100}$; 2.1 + 0.87 = 2.97

   b. $7\frac{20}{100} + 2\frac{67}{100} = 9\frac{87}{100}$; 7.2 + 2.67 = 9.87

   c. $7\frac{3}{10} + 1\frac{8}{10} = 9\frac{1}{10}$; 7.3 + 1.8 = 9.1

   d. $7\frac{30}{100} + 1\frac{86}{100} = 9\frac{16}{100}$; 7.3 + 1.86 = 9.16

   e. $6\frac{7}{100} + 3\frac{93}{100} = 10$; 6.07 + 3.93 = 10.0 or 10

   f. $6\frac{87}{100} + 3\frac{90}{100} = 10\frac{77}{100}$; 6.87 + 3.9 = 10.77

   g. $8\frac{60}{100} + 4\frac{67}{100} = 13\frac{27}{100}$; 8.6 + 4.67 = 13.27

   h. $18\frac{62}{100} + 14\frac{70}{100} = 33\frac{32}{100}$; 18.62 + 14.7 = 33.32

Lesson 14

Problem Set

1. 5.79 liters

2. 33.14 kilometers

3. 296.85 kilograms

4. 92.28 seconds

Exit Ticket

12.03 kilometers

Homework

1. 3.63 meters

2. 36.14 kilograms

3. 256.54 liters

4. 54.8 seconds

Lesson 15

Problem Set

1. $1.00; 100

2. $0.01; 1

3. $0.06; 6

4. $0.10; 10

5. $0.26; 26

6. $1.00; 10

7. $0.10; 1

8. $0.30; 3

9. $0.50; 5

10. $0.60; 6

11. $1.00; 100

12. $0.25; 25

13. $0.50; 50

14. $0.75; 75

15. $\frac{38}{100}$ dollar; $0.38

16. $\frac{103}{100}$ or $1\frac{3}{100}$ dollars; $1.03

17. $\frac{110}{100}$ or $1\frac{10}{100}$ dollar; $1.10

18. $\frac{236}{100}$ or $2\frac{36}{100}$ dollar; $2.36

19. $6.67

20. $5.35

21. $14.81

Exit Ticket

1. $\frac{80}{100}$ dollar; $0.80

2. $\frac{118}{100}$ or $1\frac{18}{100}$ dollars; $1.18

3. $6.19

Homework

1. $1.00; 100

2. $0.01; 1

3. $0.03; 3

4. $0.20; 20

5. $0.37; 37

6. $1.00; 10

7. $0.20; 2

8. $0.40; 4

9. $0.60; 6

10. $0.90; 9

11. $0.75; 75

12. $0.50; 50

13. $1.00; 100

14. $0.25; 25

15. $\frac{58}{100}$ dollar; $0.58

16. $\frac{88}{100}$ dollar; $0.88

17. $\frac{161}{100}$ or $1\frac{61}{100}$ dollar; $1.61

18. $\frac{187}{100}$ or $1\frac{87}{100}$ dollar

19. $4.08

20. $5.26

21. $16.85

Lesson 16

Sprint

Side A

1.	0.1	12.	0.79	23.	0.2	34.	1.3
2.	0.01	13.	0.79	24.	0.20	35.	1.37
3.	0.11	14.	0.4	25.	0.40	36.	1.2
4.	0.3	15.	0.06	26.	0.3	37.	1.29
5.	0.03	16.	0.46	27.	0.30	38.	1.2
6.	0.33	17.	0.64	28.	0.60	39.	1.24
7.	0.5	18.	0.85	29.	0.70	40.	1.25
8.	0.05	19.	0.92	30.	0.90	41.	1.61
9.	0.55	20.	0.81	31.	1.0	42.	1.53
10.	0.7	21.	0.14	32.	1.1	43.	1.72
11.	0.09	22.	0.47	33.	1.3	44.	1.88

Side B

1.	0.2	12.	0.48	23.	0.1	34.	1.20
2.	0.02	13.	0.48	24.	0.10	35.	1.27
3.	0.22	14.	0.5	25.	0.2	36.	1.2
4.	0.4	15.	0.07	26.	0.4	37.	1.29
5.	0.04	16.	0.57	27.	0.40	38.	1.4
6.	0.44	17.	0.57	28.	0.8	39.	1.46
7.	0.6	18.	0.96	29.	0.8	40.	1.48
8.	0.06	19.	0.83	30.	0.9	41.	1.51
9.	0.66	20.	0.71	31.	1.0	42.	1.63
10.	0.4	21.	0.13	32.	1.1	43.	1.74
11.	0.08	22.	0.38	33.	1.3	44.	1.64

Problem Set

1. $4.11
2. $3.59
3. No; $0.96
4. $9.16
5. $2.64

Exit Ticket

$2.16

Homework

1. $4.59
2. $2.20
3. No; $0.61
4. $14.97
5. $2.17